LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

CAPRICORN NORTH DISTRICT

GRADE 12

MATHEMATICS P2

DATE OF ISSUE: 12 MARCH 2024

LEVEL 1, 2, 3 & 4 SOLUTIONS MANUAL

LIMPOPO

PROVINCIAL GOVERNMENT

REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF

EDUCATION

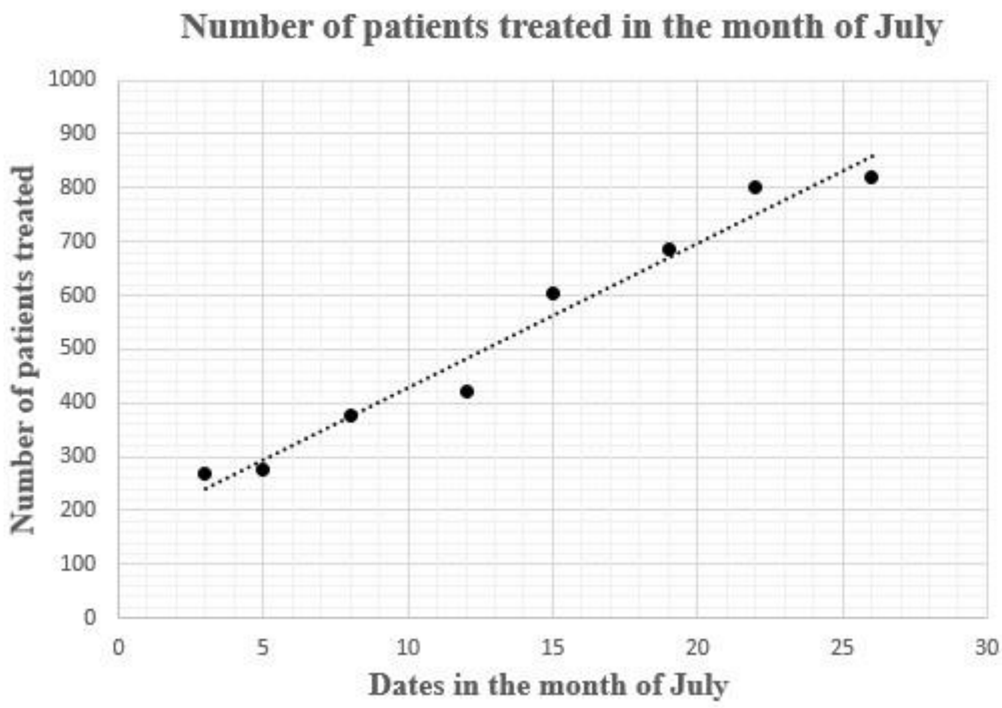
GRADE 12

MATHEMATICS

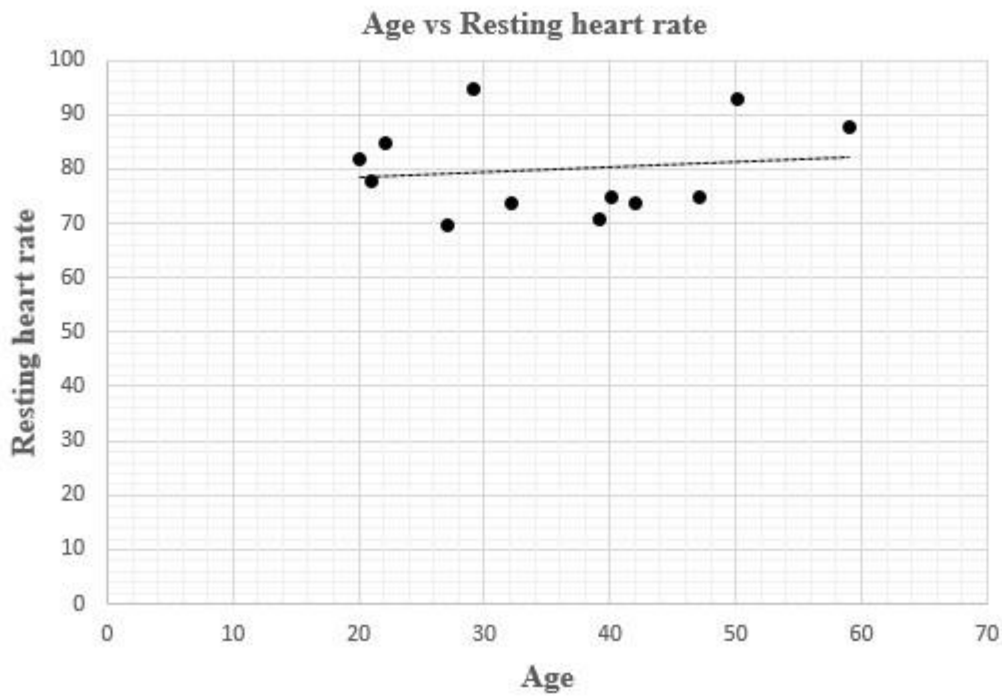
STATISTICS MARKING GUIDELINE

LEVEL 1 AND 2

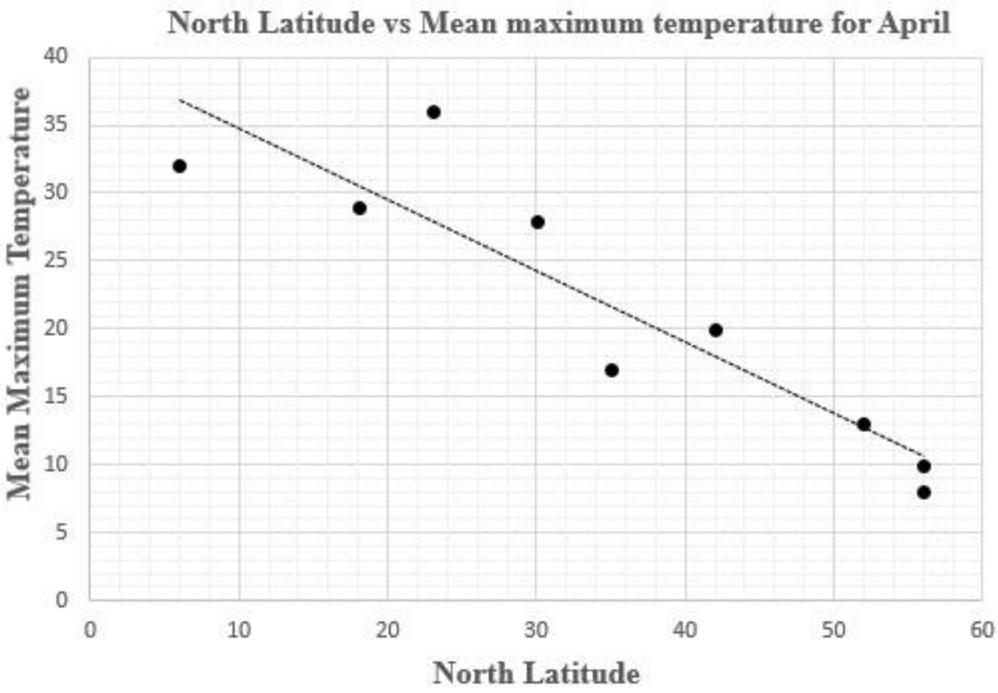
Question 1

No.	Solutions	Marks
1.1 & 1.3		(3) & (2)
1.2	$a = 161{,}24$ $b = 26{,}88$ $y = 161{,}24 + 26{,}88x$	(4)
1.4	On 30 June: $x = 0$ $y = 161{,}24 + 26{,}88(0)$ $y = 161$	(2)
1.5	On 24 July: $x = 24$ $y = 161{,}24 + 26{,}88(24)$ $y = 806{,}36$ $\therefore y = 806$	(2)
1.6	$r = 0{,}98$ There is a very strong positive correlation between the number of days elapsed in July and the number of patients that were treated. This would suggest that there was a rapid spread of the influenza virus in the community.	(3)
		[16]

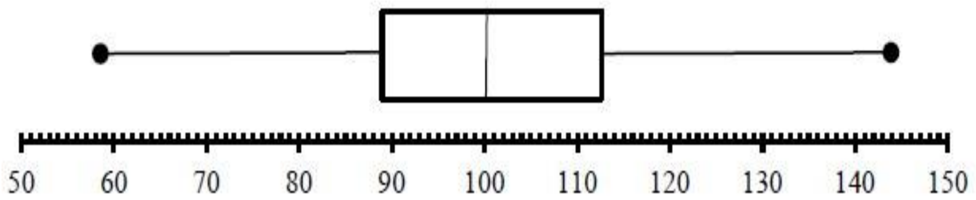
Question 2

No.	Solutions	Marks
2.1 & 2.3		(3) & (2)
2.2	$a = 76{,}60$ $b = 0{,}10$ $y = 76{,}60 + 0{,}10x$	(4)
2.4	$r = 0{,}14$	(2)
2.5	The value of r is positive and close to zero. This suggests that there is an insignificant relationship between age and resting heart rate.	(2)
2.6	No. The value of r being close to zero suggests that it is not reliable to predict the resting heart rate of a person by using age alone.	(2)
		[15]

Question 3

No.	Solutions	Marks
3.1 & 3.3		(3) & (2)
3.2	$a = 39{,}94$ $b = -0{,}52 \quad y =$ $39{,}94 - 0{,}52x$	(4)
3.4	The y-intercept represents the mean maximum temperature for April at the equator.	(1)
3.5	$y = 39{,}94 - 0{,}52x$ $y = 39{,}94 - 0{,}52(40)$ $y = 19{,}14°\text{C}$	(2)
3.6	$r = -0{,}91$	(2)
3.7	The value of r is close to – 1 and suggests that there is a very strong relationship between distance from the equator and the mean maximum temperature for April. The further one moves away from the equator, the colder it gets.	(1)
		[15]

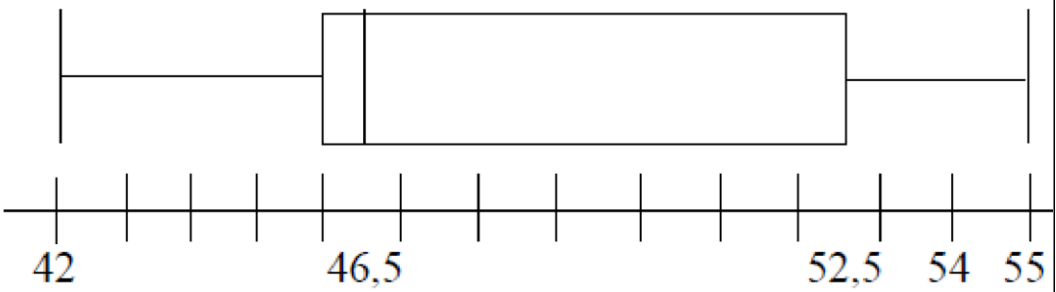
Question 4

4.1	$\bar{x} = \dfrac{1\ 522}{15}$ $\bar{x} = 101{,}7$	(2)
4.2	σ=19,07	(2)
4.3	$Lower\ quartile = 89$ $Upper\ quartile = 113$	(2)
4.4		(3)
4.5	$(\bar{x} - 1\sigma\ ;\ \bar{x} - 1\sigma\ ) = (82{,}4\ ;\ 120{,}54)$ ∴ 2 days	(3)
		[12]

Question 5

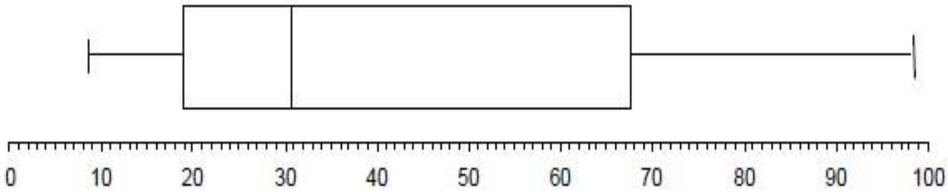
No.	Solutions	Marks
5.1	$\bar{x} = \dfrac{102\ 100}{9}$ $\bar{x} = 11\ 344{,}44$	(2)
5.2	$\sigma = 4\ 460{,}97$	(2)

Question 6

No.	Solutions	Marks
6.1	$\frac{55+55+50+47+42+3x}{8} = 48{,}375$ $\frac{249+3x}{8} = 48{,}75$ $x = 46$	(2)
6.2		(4)
		[6]

Question 7

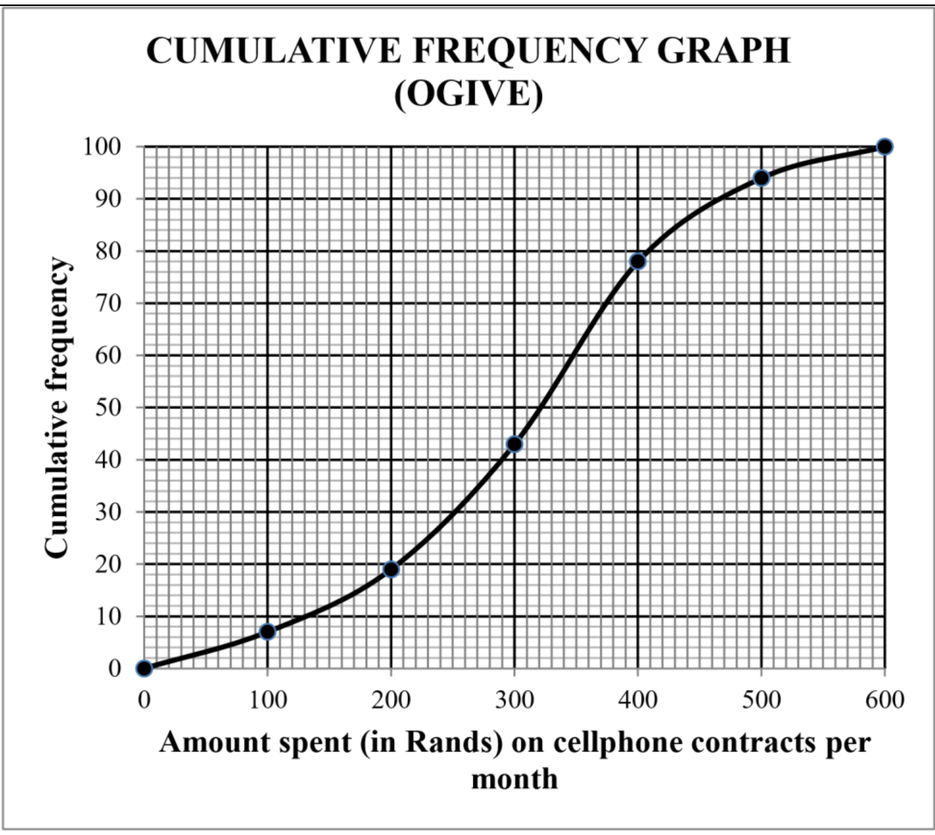
No.	Solutions	Marks
7.1	$\bar{x} = \frac{522{,}5}{12}$ $\bar{x} = 43{,}5$	(2)
7.2	$Minimum = 9{,}3$ $Lower\ quartile = \frac{+\ 23{,}6}{2} \rightarrow 19{,}3$ $Median = \frac{28+32{,}5}{2} \rightarrow 30{,}3$ $Upper\ quartile = \frac{,7+71{,}9}{2} \rightarrow 68{,}8$	(5)

	$Maximum = 98{,}2$	
7.3		(3)
7.4	The data is skewed to the right (positively skewed). This suggests that there was a large difference between the median and the maximum rainfall (some months had exceptionally high rainfall in that year).	(2)
7.5	$\sigma = 28{,}19$	(3)
		[15]

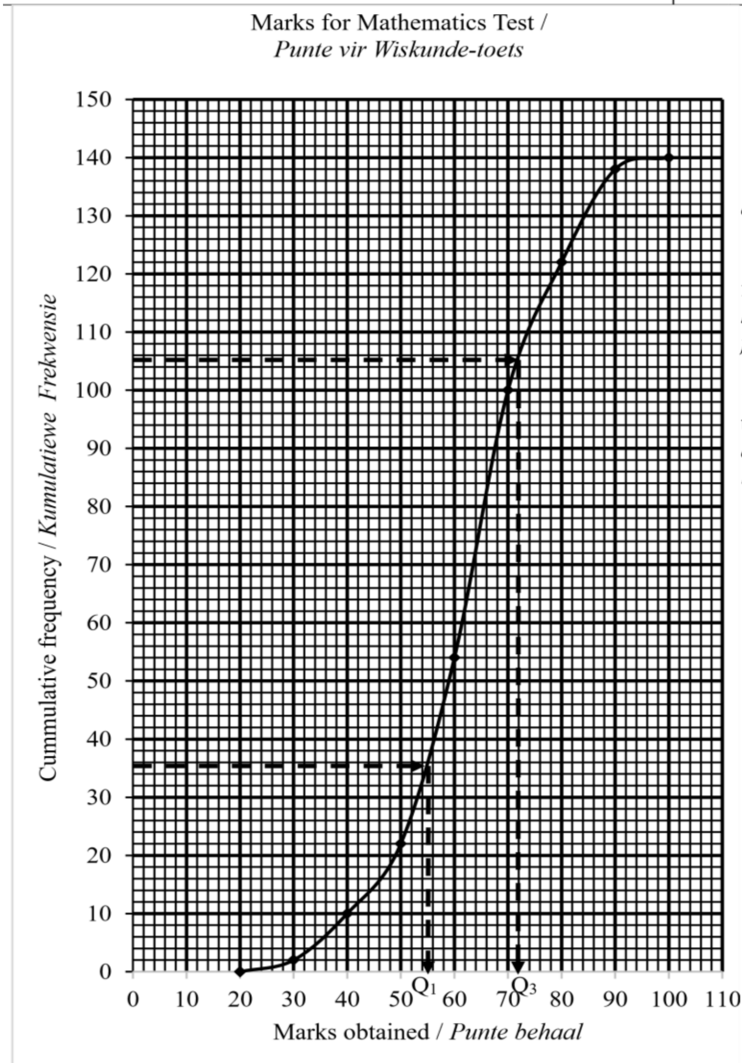
Question 8

8.1	60 employees	✓ answer (A) (1)
8.2	$20 < x \leq 25$	✓ answer (1)
8.3	60 – 34 = 26 employees ANSWER ONLY: Full marks	✓ 34 ✓ answer (2)
8.4	Salary $= \frac{100}{7} \times 2400$ Salary = R34 285,71 ANSWER ONLY: Full marks	✓ method ✓ answer (2)
		[8]

Question 9

9.1	Number people paid R200 or less = 19 *Aantal mense wat R200 of minder betaal het = 19*	✓ answer (1)
9.2	$7+12+a+35+b+6=100$ $a=40-b$ $309=\dfrac{(50\times 7)+(150\times 12)+(250\times a)+(350\times 35)+(450\times b)+(550\times 6)}{100}$ $309=\dfrac{(50\times 7)+(150\times 12)+(250\times (40-b))+(350\times 35)+(450\times b)+(550\times 6)}{100}$ $350 + 1800 + 10000 - 250b + 12250 + 450b + 3300 = 30900$ $200b = 3200$ $b = 16$ $a = 24$	(5) (5)
9.3	Modal class*:* $300 < x \leq 400$	✓ answer (1)
9.4		(4)
9.5	Number of people *mense* = 100 – 82 [accept 80 – 84 people] 18 people paid more than R420 per month/. [accept 16 – 20 people] Answer only: Full marks	✓ 82 ✓ answer (2)
		[13]

QUESTION 10

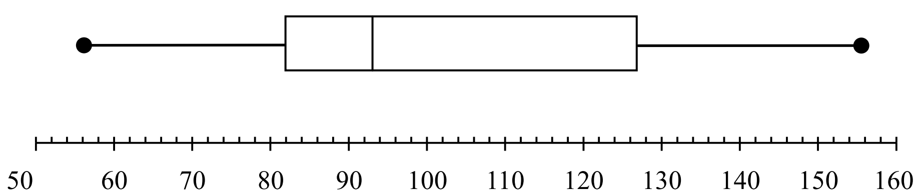
140 learners / *leerders*	✓answer / *antwoord*	(1)
$60 < x \leq 70$	✓answer / *antwoord*	(1)
	✓ grounding / *anker* ✓cumulative frequency / *kumulatiewe frekwensie* ✓plotting against the upper limit / *afsteek teen die boonste limiet* ✓shape / *vorm*	(4)
$Q_3 - Q_1 = 72 - 55$ $= 17$	✓ Q_3 & Q_1 ✓ IQR	(2)
		[8]

Question 11

11.1.1	$\bar{x} = \frac{396}{18}$ $= 22$	✓ 396 ✓ answer (2)
11.1.2	σ= 10,1707≈ 10,17	✓ answer (1)
11.2	22×18=396 ordered 20×18 = 360sold Total not sold is 36	(2)
11.3.1	Option B <u>Any one of the following reasons</u>Median/ = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right	✓ B ✓ reason (2)
11.32	Data is positively skewed/skewed to the right	✓ answer (1)
		[10]

QUESTION 12

56	68	69	71	71	72	82	84	85
88	89	90	92	93	94	96	97	99
102	103	127	128	134	135	137	144	156

12.1	Range/*Omvang* = 156 – 56 = 100 kg	✓ max – min ✓ answer/(2)
12.2	Mode/*Modus* = 71 kg	✓ answer (1)
12.3	Median/*Mediaan* = T_{14} = 93 kg	✓ answer/ (1)
12.4	$Q_1 = T_7 = 82$ $Q_3 = T_{21} = 127$ IQR $= Q_3 - Q_1$ $= 127 - 82$ $= 45$ kg	✓ $Q_1 = 82$ ✓ $Q_3 = 127$ ✓ answer (3)
12.5		✓ box ✓ whiskers (2)
12.6	SD = 25,838 ≈ 25,84 kg	✓✓ answer (2)

QUESTION 13

13.1	Mean / *Gemiddelde* $= \frac{25+47+40+34+28+x+37+28+55+30}{10}$ $= \frac{324+x}{10}$	✓answer / *antwoord* (1)
13.2	$\frac{324+x}{10} = 36$ $x = 36$	✓equating / *gelykstel* ✓answer / *antwoord* (2)
13.3	8,88	✓✓answer / *antwoord* (2)

QUESTION 14

14.1.1	175
14.1.2	$40 \le x < 50$ **OR** $40 < x \le 50$
14.1.3	175 – 158 = 17
14.2.1	$\bar{x} = 74,87$
14.2.2	$\sigma = 16,12$

14.3	$\bar{x}-\sigma=82,7$ $\bar{x}+\sigma=94,1$ $2\bar{x}=176,8$ $\bar{x}=88,4$ $\sigma=88,4-82,7$ OR $\sigma=94,1-88,4$ $\sigma=5,7$ $\sigma=5,7$ **OR** $\bar{x}=\frac{82,7+94,1}{2}$ $\bar{x}=88,4$ $\sigma=88,4-82,7$ OR $\sigma=94,1-88,4$ $\sigma=5,7$ $\sigma=5,7$	✓✓ $\bar{x}=88,4$ ✓ answer (3) ✓✓ $\bar{x}=88,4$ ✓ answer (3)	
		[12]	

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MATHEMATICS

STATISTIC MARKING GUIDELINE

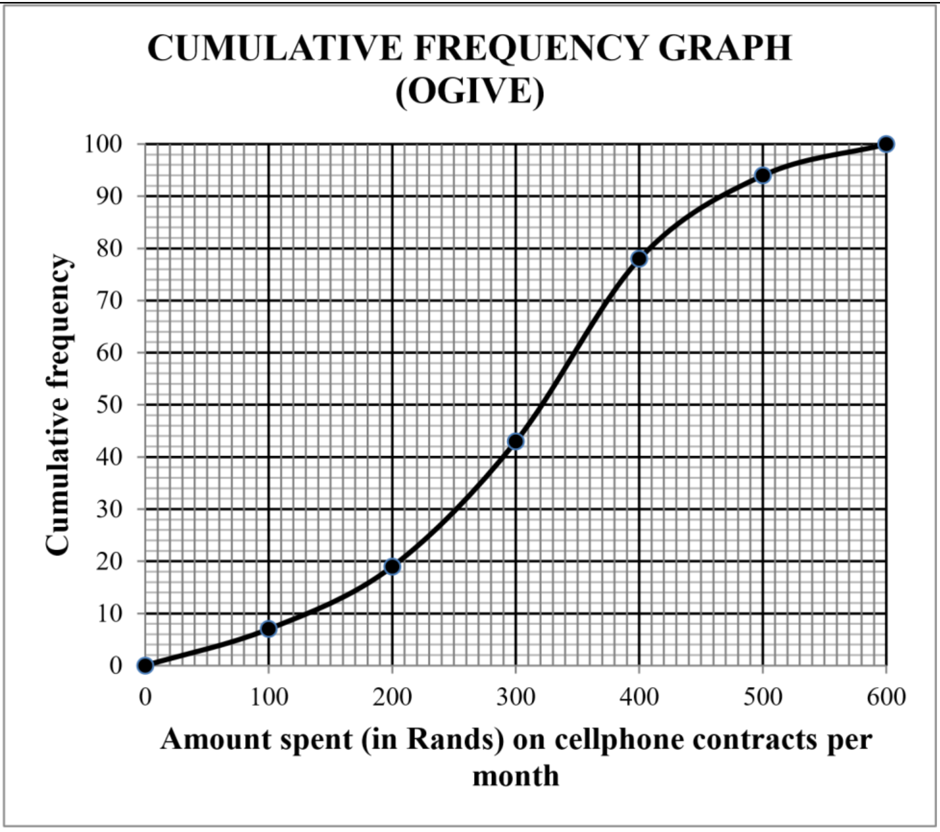
LEVEL 3 AND 4

Question 1

1.1	60 employees	✓ answer (A) (1)
1.2	$20 < x \leq 25$	✓ answer (1)
1.3	60 – 34 = 26 employees ANSWER ONLY: Full marks	✓ 34 ✓ answer (2)
1.4	Salary $= \frac{100}{7} \times 2400$ Salary = R34 285,71 ANSWER ONLY: Full marks	✓ method ✓ answer (2)
1.5	∴Ogive/Cumulative frequency graph will shift to the right/will become steeper.	✓✓ answer (2)
		[8]

Question 2

2.1	Number people paid R200 or less = 19 *Aantal mense wat R200 of minder betaal het = 19*	✓ answer (1)
2.2	$7+12+a+35+b+6=100$ $a=40-b$ $309=\frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times b)+(550\times6)}{100}$ $309=\frac{(50\times7)+(150\times12)+(250\times(40-b))+(350\times35)+(450\times b)+(550\times6)}{100}$ $350 + 1800 + 10000 - 250b + 12250 + 450b + 3300 = 30900$ $200b = 3200$ $b = 16$ $a = 24$	(5) (5)
2.3	Modal class*:* $300 < x \leq 400$	✓ answer (1)

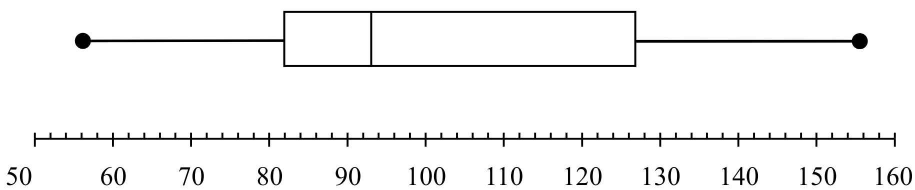
2.4		(4)
2.5	Number of people *mense* = 100 – 82 [accept 80 – 84 people] 18 people paid more than R420 per month/. [accept 16 – 20 people] Answer only: Full marks	✓ 82 ✓ answer (2)
		[13]

Question 3

3.1.1	$\bar{x} = \frac{396}{18}$ $= 22$	✓ 396 ✓ answer (2)
3.1.2	$\sigma = 10{,}1707 \approx 10{,}17$	✓ answer (1)
3.1.3	$\bar{x} + \sigma = 32{,}17$ $\therefore 5$ days	✓ 32,17 ✓ 5 (2)
3.2	$22 \times 18 = 396$ ordered $20 \times 18 = 360$sold Total not sold is 36	(2)
3.3.1	Option B Any one of the following reasonsMedian/ = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right	✓ B ✓ reason (2)
3.32	Data is positively skewed/skewed to the right	✓ answer (1)
		[10]

QUESTION 4

56	68	69	71	71	72	82	84	85
88	89	90	92	93	94	96	97	99
102	103	127	128	134	135	137	144	156

4.1	Range/*Omvang* = 156 – 56 = 100 kg	✓ max – min ✓ answer/(2)
4.2	Mode/*Modus* = 71 kg	✓ answer (1)
4.3	Median/*Mediaan* = T_{14} = 93 kg	✓ answer/ (1)
4.4	$Q_1 = T_7 = 82$ $Q_3 = T_{21} = 127$ IQR $= Q_3 - Q_1$ $= 127 - 82$ $= 45$ kg	✓ $Q_1 = 82$ ✓ $Q_3 = 127$ ✓ answer (3)
4.5		✓ box ✓ whiskers (2)
4.6	SD = 25,838 ≈ 25,84 kg	✓✓ answer (2)
4.7	$\bar{x} = 98,59$ $\bar{x} + 1\sigma = 98,59 + 25,84$ $= 124,43$ kg 127 > 124,43 ∴ I agree with this person.	✓ $\bar{x} = 98,59$ ✓ 124,43 ✓ conclusion/ (3) **[14]**

QUESTION 5

5.1	Mean / *Gemiddelde* $= \frac{25+47+40+34+28+x+37+28+55+30}{10}$ $= \frac{324+x}{10}$	✓answer / *antwoord* (1)
5.2	$\frac{324+x}{10} = 36$ $x = 36$	✓equating / *gelykstel* ✓answer / *antwoord* (2)
5.3	8,88	✓✓answer / *antwoord* (2)
5.4	outside [36 – 8,88 ; 36 + 8,88] = [27,12 ; 44,88] ∴ 3 people	✓method / *metode* ✓answer / *antwoord* (2) [7]

QUESTION 6

6.1.1	175	
6.1.2	$40 \leq x < 50$ **OR** $40 < x \leq 50$	
6.1.3	$175 - 158 = 17$	
6.2.1	$\bar{x} = 74,87$	
6.2.2	$\sigma = 16,12$	
6.2.3	$\bar{x}+\sigma = 74,87+16,12 = 90,99$ 3 learners	
6.3	$\bar{x}-\sigma=82,7$ $\bar{x}+\sigma=94,1$ $2\bar{x}=176,8$ $\bar{x}=88,4$ $\sigma=88,4-82,7$ OR $\sigma=94,1-88,4$ $\sigma=5,7$ $\sigma=5,7$ **OR** $\bar{x}=\frac{82,7+94,1}{2}$ $\bar{x}=88,4$ $\sigma=88,4-82,7$ OR $\sigma=94,1-88,4$ $\sigma=5,7$ $\sigma=5,7$	✓✓ $\bar{x}=88,4$ ✓ answer (3) ✓✓ $\bar{x}=88,4$ ✓ answer (3)
		[12]

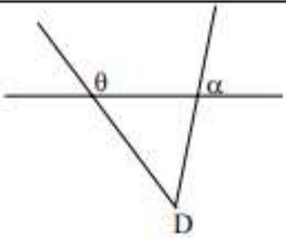
MARKING GUIDELINES: LINES , POLYGONS AND CIRCLES

PART A: QUESTION 1 -10: LINES AND POLYGONS

PART B: QUESTION 1 – 10: CIRCLES

PART A

QUESTION 1

1.1	$AC=\sqrt{(2+4)^2+(5-3)^2}$ $AC=\sqrt{40}$ $AC=2\sqrt{10}$	✓ substitution ✓ answer (2)
1.2	$M(\frac{-4+2}{2};\frac{3+5}{2})\quad \therefore M(-1;4)$	✓ substitution ✓ answer (2)
1.3	$m_{BD}=\frac{10-(-2)}{-3-1}=\frac{12}{-4}=-3$ $m_{AC}=\frac{5-3}{2-(-4)}=\frac{2}{6}=\frac{1}{3}$ $\therefore m_{BD}\times m_{AC}=-3\times\frac{1}{3}=-1$ $\therefore BD\perp AC$ $Midpoint\ BD\left(\frac{-3+1}{2};\frac{10-2}{2}\right)=Midpoint\ of\ AC$ $=(-1;4)$ $\therefore$ bisect at $90°$	✓ answer ✓ answer ✓ -1 ✓ coordinates ✓ = Midpoint AC (5)
1.4	Area ΔABC $=\frac{1}{2}$AC.MB $=\frac{1}{2}.\sqrt{40}.\sqrt{(10-4)^2+(-3+1)^2}$ $=\frac{1}{2}\sqrt{40}.\sqrt{40}$ $=20$	✓ formula ✓ substitution ✓ MB = $\sqrt{40}$ ✓answer (4)
1.5	$m_{DC}=\frac{3+2}{-4-1}=-1$ $y+2=-1(x-1)$ $y=-x-1$	✓ substitution ✓ answer ✓ answer (3)
1.6	$m_{DC}=-1$ $\tan\theta=-1$ $\theta=135°$	✓ substitution ✓ answer (2)
1.7	$m_{AD}=\frac{5+2}{2-1}=7$ $\therefore\tan\alpha=7$ $\alpha=81{,}9°$ $A\hat{D}C=\theta-\alpha$ $A\hat{D}C=135°-81{,}9°=53{,}1°$ OR  	✓ 7 ✓ 81,9° ✓ $A\hat{D}C=\theta-\alpha$ ✓ 53,1° (4)

NSC
MEMORANDUM

	$A\hat{D}C = 180° - (45° + 81,9°)$ $A\hat{D}C = 53,1°$ OR999 Use Cosine Rule $AC^2 = DC^2 + AD^2 - 2DC.AD\cos D$ $\therefore 40 = 50 + 50 - 2 \times 50\cos D$ $\therefore \cos D = 0.6 \quad \therefore \hat{D} = 53,13°$	✓ Use Cosine Rule ✓✓Substitution ✓ answer **[22]**

QUESTION 2

2.1	$m_{BC} = \frac{1-0}{6-3}$ $m_{BC} = \frac{1}{3}$	✓ substitution into gradient formula ✓ answer (2)
2.2.	$m_{AD} = m_{BC}$ $m_{AD} = \frac{1}{3}$ ----------------------------- AB//BC $\therefore$ Equation of AD is: $y = \frac{1}{3}x + c$ $6 = \frac{1}{3}(1) + c$ $c = \frac{17}{3}$ $\therefore y = \frac{1}{3}x + \frac{17}{3}$ **OR** $y - 6 = \frac{1}{3}(x-1)$ $y - 6 = \frac{1}{3}x - \frac{1}{3}$ $y = \frac{1}{3}x + \frac{17}{3}$	✓ $m_{AC} = \frac{1}{3}$ ✓ substitution of (1 ; 6) into a straight line equation ✓equation (3) ✓ $m_{AC} = \frac{1}{3}$ ✓ substitution of (1 ; 6) into a straight line equation ✓equation (3)
1.3 2.3.	$y = \frac{1}{3}x + \frac{17}{3}$ $t = \frac{1}{3}(7) + \frac{17}{3}$ $t = 8$ **OR** $\frac{t-6}{7-1} = \frac{1}{3}$ $t - 6 = 2$ $\therefore t = 8$	✓ ✓substitution of x value into a straight line equation. (2)

2.4.	$AD=\sqrt{(8-6)^2+(-1-3)^2}$ $AD=\sqrt{40}$ $AD=2\sqrt{10}$ $BC=\sqrt{(6-3)^2+(1-0)^2}$ $BC=\sqrt{10}$ $AB=\sqrt{(6-0)^2+(1-3)^2}$ $AB=\sqrt{40}$ $AB=2\sqrt{10}$	✓ using distance formula ✓ answer for AD ✓ answer for BC ✓ answer for AB (4)
2.5	$m_{AB}=\frac{6-0}{1-3}$ $m_{AB}=-3$ $m_{BC}=\frac{1-0}{6-3}=\frac{1}{3}$ $m_{AB}.m_{BC}=\frac{1}{3}\times-3$ $=-1$ $\therefore AB\perp BC$	✓ $m_{AB}=-3$ ✓ $m_{AB}\times m_{BC}=-1$ ✓ conclusion (3)
2.6	Area of Quad ABCD = area of ΔADC + area of ABC $=\frac{1}{2}(2\sqrt{10})(2\sqrt{10})+\frac{1}{2}(\sqrt{10})(2\sqrt{10})$ $=20+10$ $=30$ square units Or Area of ABCD $=\frac{1}{2}(\text{sum of parallel sides})\times h$ $=\frac{1}{2}(2\sqrt{10}+\sqrt{10})2\sqrt{10}$ $=\sqrt{10}(3\sqrt{10})$ $=30$ square units	✓ formula for area of Δ ✓✓ $\frac{1}{2}(2\sqrt{10})(2\sqrt{10})+\frac{1}{2}(\sqrt{10})(2\sqrt{10})$ ✓ answer (4) ✓ formula for area of trapezium ✓✓ $\frac{1}{2}(2\sqrt{10}+\sqrt{10})2\sqrt{10}$ ✓ Answer (4)
2.7	From 1.1 $m_{BC}=\frac{1}{3}$ $\tan\theta=\frac{1}{3}$ $\therefore\theta=18,43°$	✓✓ $\tan\theta=\frac{1}{3}$ ✓ $\theta=18,43°$ (3) **[21]**

QUESTION 3

3.1	AB is defined as $5y-3x-5=0$ which can be written as $y=\frac{3}{5}x+1$ $m_{AB}=\frac{3}{5}$ Let α be the inclination of AB. $\tan\alpha=\frac{3}{5}$ $\alpha=30.96°$. Let β be the inclination of CD $\beta=45°+30{,}96°$ $=75{,}96°$ Gradient of CD = tan 75,96° = 4. **OR** $\tan\beta=\tan(\alpha+45°)$ $=\frac{\tan\alpha+\tan 45°}{1-\tan\alpha.\tan 45°}$ $=\frac{\frac{3}{5}+1}{1-\frac{3}{5}\times 1}$ $=4$ $m_{CD}=\tan\beta$ $m_{CD}=4$	✓ $m_{AB}=\frac{3}{5}$ ✓ $\tan\alpha=\frac{3}{5}$ ✓ $\alpha=30.96°$ ✓ $\beta=75{,}96°$ ✓ gradient of CD (5) ✓ expansion ✓ tan 45° = 1 ✓ $\tan\alpha=\frac{3}{5}$ ✓ substitution ✓ answer (5)

3.2	Equation of CD is $y=4x+c$ $\therefore\ 4=4(5)+c$ $c=-16$ Equation of CD is $y=4x-16$. **OR** $y-4=4(x-5)$ $y-4=4x-20$ $y=4x-16$	✓ y- intercept ✓ equation of CD (2) ✓ substitution ✓ equation of CD (2) **[7]**

QUESTION 4

4.1.1.	$m_{PT} = \tan\ 63{,}43°$ $= 2$	✓ tan 63,43° ✓ 2 (2) Answer only: full marks
4.1.2	Coordinates of P(– 5 ; 0) $y - y_1 = m(x - x_1)$ $y - 0 = 2(x+5)$ $y = 2x + 10$ **OR** $y = mx + c$ $0 = (2)(-5) + c$ $c = 10$ $y = 2x + 10$ **OR** $m_{PT} = 2 = \tan 63{,}43°$ $\tan 63{,}43° = \frac{\text{OS}}{\text{OP}} = \frac{\text{OS}}{5} = 2$ $\therefore \text{OS} = 10$ $y = 2x + 10$	✓substitution of P(–5 ; 0) and $m = 2$ into equation ✓ equation (2) ✓substitution of P(–5 ; 0) and $m = 2$ into equation ✓ equation (2) ✓ $\frac{\text{OS}}{5} = 2$ ✓ equation (2)

4.1.3	OS = 10 units $PS^2 = (5)^2 + (10)^2$ $= 125$ $PS = \sqrt{125} = 5\sqrt{5}$ Accept PS = 11,18	✓ OS = 10 ✓ substitution of correct distances into Pythagoras ✓ $\sqrt{125}$ (3)
	OR	
	P(−5 ; 0) ; OS = 10 units $PS^2 = (-5-0)^2 + (0-10)^2$ $= 25 + 100$ $= 125$ $PS = \sqrt{125} = 5\sqrt{5}$ Accept PS = 11,18	✓ OS = 10 ✓ substitution of correct distances into Pythagoras ✓ $\sqrt{125}$ (3)
	OR	
	$\frac{PS}{5} = \frac{1}{\cos 63{,}43°}$ $\therefore PS = \frac{5}{\cos 63{,}43°}$ $PS = 11{,}18$	✓ ratio ✓ $PS = \frac{5}{\cos 63{,}43°}$ ✓11,18 (3)
	OR	
	$\frac{PS}{10} = \frac{1}{\sin 63{,}43°}$ $\therefore PS = \frac{10}{\sin 63{,}43°}$ $PS = 11{,}18$	✓ ratio ✓ $PS = \frac{10}{\sin 63{,}43°}$ ✓11,18 (3)

4.1.4	Let T be $(x ; y)$. Then $\frac{-5+x}{2} = 0$ and $\frac{0+y}{2} = 10$ $x = 5$ $y = 20$ T(5 ; 20)	✓ 5 ✓ 20 (2)
	OR by inspection: T(5 ; 20)	✓ 5 ✓ 20 (2)
4.2	$OR = \left(\frac{3}{2}\right)(5) = \frac{15}{2} = 7{,}5$ $R\left(\frac{15}{2}; 0\right)$ If only x-coordinate : 2 marks	✓$x = 7{,}5$ / $\frac{15}{2}$ ✓$y = 0$ (2)

4.3	Area $\Delta PTR = \frac{1}{2}(\text{base PR}) \times (\text{height})$ $= \frac{1}{2}(5 + \frac{15}{2}) \times 20$ $= 125$ square units	✓ area formula ✓ $5 + \frac{15}{2} = 12,5$ ✓ 20 ✓ 125 (4)
	OR Area $\Delta PTR = \frac{1}{2} PT.PR.\sin T\hat{P}R$ $= \frac{1}{2}\left(10\sqrt{5}\right)\left(\frac{25}{2}\right)\sin 63,43°$ $= 124,99$ square units	✓ area formula ✓ $10\sqrt{5}$ ✓ $\frac{25}{2}$ ✓ 124,99 (4) **[15]**

QUESTION 5

5.1.1	$y = -x - 11$ $A(-1\,;\,t)$ $t = -(-1) - 11$ $t = -10$	✓ substitution ✓ value of t (2)
5.1.2	$\tan\alpha = -1$ $ref.\angle = 45°$ $\therefore \alpha = 135°$	✓ $\tan\alpha = -1$ ✓ 135° (2)

5.1.3	$\tan 63,43° = m_{AC}$ $m_{AC} = 2$	✓ $\tan 63,43° = m_{AC}$ ✓ answer (2)
5.2	$m_{AC} = 2$ $A(-1\,;\,-10)$ $y = 2x + k$ **OR/*OF*** $y - y_1 = 2(x - x_1)$ $-10 = 2(-1) + k$ $y - (-10) = 2(x - (-1))$ $k = -8$ $y = 2x - 8$ $y = 2x - 8$	✓ substitution of m and A ✓ equation (2)

5.3.1	$y=2x-8$ $0=2x-8$ $x_B=4$ $\frac{x_C+(-1)}{2}=4$ $\frac{y_C+(-10)}{2}=0$ $x_C=9$ $y_C=10$ **OR/*OF*** by translation / *met translasie* $A \to B\ (x;y) \to (x+5;y+10)$ $B \to C\ (4;0) \to (4+5;\ 0+10)=(9\ ;\ 10)$	✓ $x_B=4$ ✓ $x_C=9$ ✓ $y_C=10$ (3) ✓ $(x+5;y+10)$ ✓ $x_C=9$ ✓ $y_C=10$ (3)
5.3.2.	$A\hat{B}E=63,43°$ [vert. opp ∠'s =] $\hat{E}_2=63,43°$ [corres. ∠'s, DE ∥ AB] $\hat{E}_1=45°$ [∠s on a str line] $F\hat{E}D=108,43°$ **OR/*OF*** $E\hat{A}B=135°-63,43°$ $E\hat{A}B=71,57°$ $D\hat{E}A=E\hat{A}B=71,57°$ $F\hat{E}D=108,43°$	✓ $A\hat{B}E=63,43°$ ✓ $\hat{E}_1=45°$ ✓ $F\hat{E}D=108,43°$ (3) ✓ $E\hat{A}B=71,57°$ ✓ $D\hat{E}A=E\hat{A}B=71,57°$ ✓ $F\hat{E}D=108,43°$ (3)
5.4	$y=0$ $x_E=-11$ $\frac{x_G+(-11)}{2}=4$ $x_G=19$ $(x-19)^2+y^2=15^2$ $(x-19)^2+y^2=225$	✓ $x_E=-11$ ✓ $x_G=19$ ✓ $(x-19)^2+y^2$ ✓ 225 (4)
		[18]

QUESTION 6

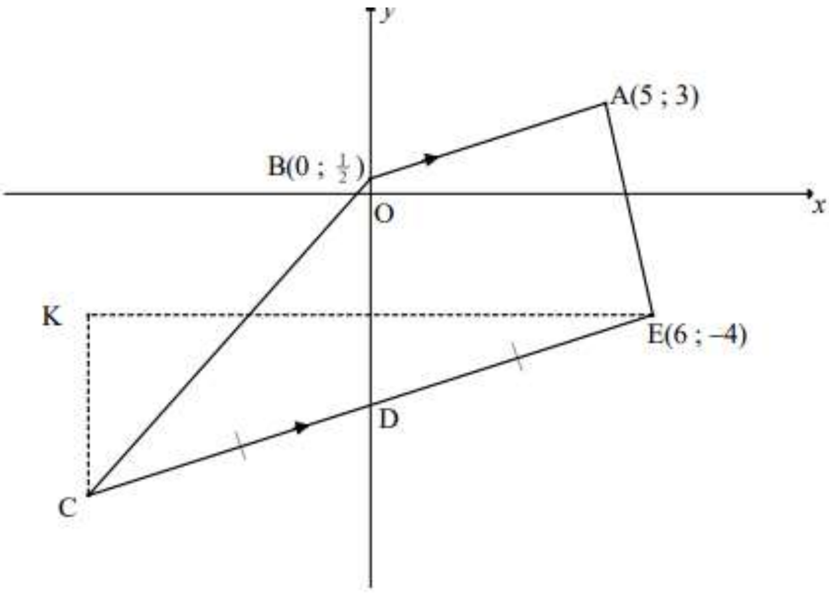
6.1.1	$m_{BE} = m_{CE} = \frac{0-(-2)}{12-4}$ **OR/*OF*** $m_{BE} = m_{CE} = \frac{-2-0}{4-12}$ $= \frac{1}{4}$ $\quad\quad$ $= \frac{1}{4}$	✓ substitution C & E ✓ answer (2)
6.1.2	$m_{AB} = \tan 81{,}87°$ $m_{AB} = 7$ Answer only: Full marks *Slegs antw: Volpunte*	✓ substitution ✓ answer (2)

6.2	$y = mx + c$ or $y - y_1 = m(x - x_1)$ $0 = \frac{1}{4}(12) + c$ or $y - 0 = \frac{1}{4}(x-12)$ $c = -3$ $\quad$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$ **OR/*OF*** $y = mx + c$ or $y - y_1 = m(x - x_1)$ $-2 = \frac{1}{4}(4) + c$ or $y - (-2) = \frac{1}{4}(x-4)$ $c = -3$ $\quad$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

6.3.1	$y = \frac{1}{4}x - 3$ $k = \frac{1}{4}k - 3$ $\frac{3}{4}k = -3$ $k = -4$ $\therefore B(-4; -4)$ **OR/*OF*** $m_{BE} = \frac{1}{4}$ **OR/*OF*** $m_{BE} = \frac{1}{4}$ $\frac{0-k}{12-k} = \frac{1}{4}$ $\quad$ $\frac{k}{k-12} = \frac{1}{4}$ $-4k = 12 - k$ $\quad$ $4k = k - 12$ $k = -4$ $\quad$ $k = -4$ $\therefore B(-4; -4)$	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2)

6.3.2	In ΔAFG: $m_{AC} = \frac{10-(-2)}{-2-4} = -2$ $\tan\theta = m_{AC} = -2$ $\theta = 180° - 63{,}43...°$ $\therefore \theta = 116{,}57°$ $\therefore \hat{A} = 116{,}57° - 81{,}87°$ [ext $\angle$ of Δ] $\therefore \hat{A} = 34{,}70°$ **OR/*OF*** In ΔABC: $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\left(2\sqrt{17}\right)^2 = \left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$ $\cos A = \frac{\left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - \left(2\sqrt{17}\right)^2}{2\left(6\sqrt{5}\right)\left(10\sqrt{2}\right)}$ $= 0{,}822...$ $\therefore A = 34{,}7°$	✓ $m_{AC} = -2$ ✓ $\tan\theta = -2$ ✓ $\theta = 116{,}57°$ ✓ answer (4) ✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)
6.3.3	$M\left(\frac{12+(-2)}{2}; \frac{10+(0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ x-value ✓ y-value (2)
6.4.1	BE = ET $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $\left(4\sqrt{17}\right)^2 = \left(\sqrt{(12-p)^2 + (0-p)^2}\right)^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ $\therefore p = 16$ or $p = -4$ (n.a.) $\therefore$T(16; 16)	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
6.4.2 a	$(x-12)^2 + y^2 = \left(4\sqrt{17}\right)^2 = 272$	✓ LHS ✓ RHS (2)
6.4.2.b	$m_{radius} = \frac{1}{4}$ $m_{tangent} = -4$ $y = -4x + c$ $-4 = -4(-4) + c$ $c = -20$ $y = -4x - 20$ **OR/*OF*** $y - y_1 = -4(x - x_1)$ $y - (-4) = -4(x - (-4))$ $y = -4x - 20$	✓ $m_{tangent}$ ✓ substitution of B ✓ equation (3)
		[24]

QUESTION 7



7.1	$m_{AB} = \dfrac{3-\frac{1}{2}}{5-0}$ $m_{AB} = \dfrac{1}{2}$ $\boxed{\text{Answer only 2/2}}$	✓ substitution ✓ answer (2)
7.2	$m_{CE} = m_{BA} = \dfrac{1}{2}$ $-4 = \dfrac{1}{2}(6)+c$ **OR/*OF*** $y-(-4) = \dfrac{1}{2}(x-6)$ $c = -7$ $y = \dfrac{1}{2}x - 7$	✓ gradient ✓ substitution of E ✓ answer (3)

7.3.1	D(0 ; –7) $\frac{x_C+6}{2}=0$ $\quad\quad$ $\frac{y_C+(-4)}{2}=-7$ $x_C=-6$ $\quad\quad$ $y_C=-10$ C(–6 ; –10) Answer only 3/3	✓ D(0 ; –7) ✓ $x_C=-6$ ✓ $y_C=-10$ (3)
7.3.2	Area Δ BCD $=\frac{1}{2}(7,5)(6)$ $=22,5$ Area Δ ABD $=\frac{1}{2}(7,5)(5)$ $=18,75$ Area ABCD $=22,5+18,75=41,25$ units^2	✓ subst of correct base and height into the area formula ✓ area ΔBCD = 22,5 ✓ area ΔABD = 18,75 ✓ answer (4)

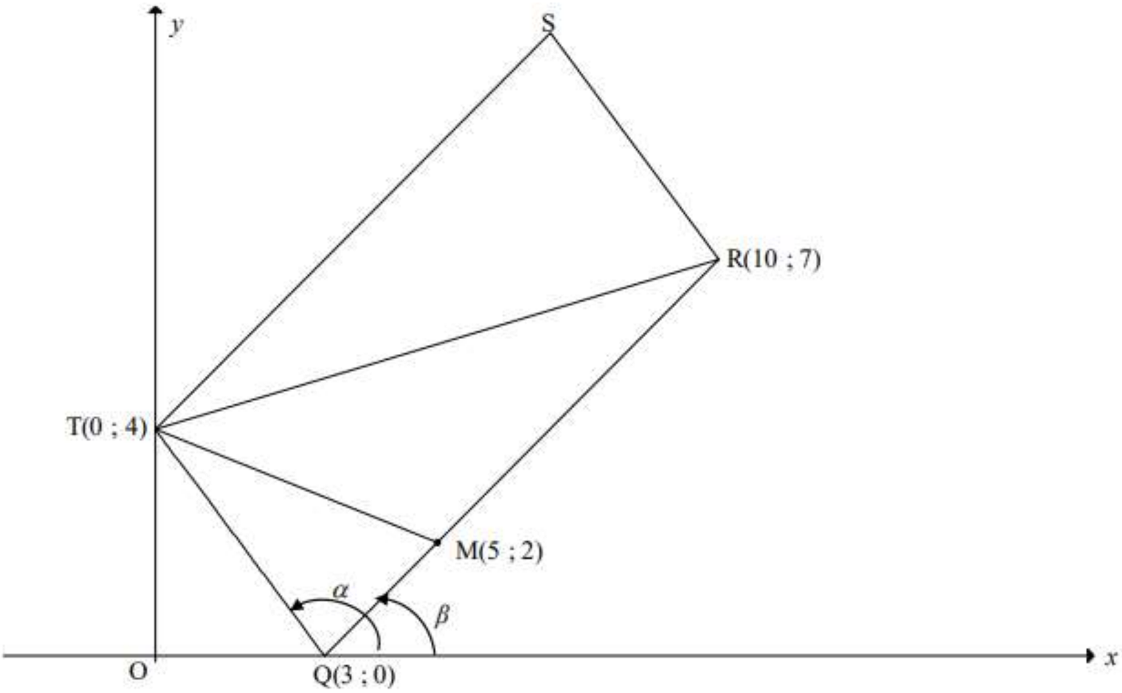
7.4.1	K(–6 ; –4)	✓ $x_K=-6$ ✓ $y_K=-4$ (2)
7.4.2a	KC = 6 units; KE = 12 units; $CE=\sqrt{(6)^2+(12)^2}$ [Pythagoras] $CE=\sqrt{180}=6\sqrt{5}=13,42$ Perimeter ΔKEC $=6+12+\sqrt{180}$ $=31,42$ units	✓ KC = 6 units ✓ KE = 12 units ✓ CE ✓ answer (4)
7.4.2b	$\tan K\hat{C}E=\frac{KE}{KC}=\frac{12}{6}=2$ $K\hat{C}E=63,43°$ **OR/*OF*** $\sin K\hat{C}E=\frac{KE}{CE}=\frac{12}{\sqrt{180}}=\frac{2\sqrt{5}}{5}$ $K\hat{C}E=63,43°$	✓ trig ratio ✓ $\tan K\hat{C}E=2$ ✓ answer (3) ✓ trig ratio ✓ $\sin K\hat{C}E=\frac{12}{\sqrt{180}}$ ✓ answer (3)

QUESTION 8

8.1	$M\left(\frac{4+8}{2};\frac{-8+0}{2}\right)$ $M(6;-4)$	✓ x_M ✓ y_M (2)
8.2	$m_{NS}=\frac{0-(-16)}{8-0}$ or $m_{NQ}=\frac{0-(-8)}{8-4}$ or $m_{QS}=\frac{-8-(-16)}{4-0}$ $=2$	✓ subst N and Q or N and Q or Q and S into gradient formula ✓ answer (2)
8.3	$m_{LQ}\times 2=-1$ [LQ⊥ NS] $\therefore m_{LQ}=-\frac{1}{2}$ $-8=-\frac{1}{2}(4)+c$ **OR** $y+8=-\frac{1}{2}(x-4)$ $c=-6$ $y+8=-\frac{1}{2}x+2$ $\therefore y=-\frac{1}{2}x-6$	✓ m_{LQ} ✓ substitution of Q ✓ calculation of c or simplification (3)
8.4	OS is the radius of circle passing through S $(x-0)^2+(y-0)^2=(16)^2$ $x^2+y^2=256$ Answer only: Full marks	✓ identifying radius = 16 ✓ Equation of circle (2)

8.5	$m_{RM} = m_{LQ} = -\frac{1}{2}$ [RM ∥ LQ] $-4 = -\frac{1}{2}(6) + c$ **OR** $y + 4 = -\frac{1}{2}(x-6)$ $c = -1$ $y + 4 = -\frac{1}{2}x + 3$ $\therefore y = -\frac{1}{2}x - 1$ T(0;−1)	✓ m_{RM} ✓ substitution of M(6;−4) ✓ coordinates of T (3)
8.6	T(0;−1), P(0;−6) and S(0;−16) ∴ PS = 10 units and TS = 15 units $\frac{LS}{RS} = \frac{PS}{TS} = \frac{2}{3}$ [prop theorem; RM ∥ LP] **OR** [line ∥ one side of Δ/*lyn ∥ een sy v Δ*] Answer only: Full marks **OR** M(6 ; –4), Q(4 ; –8) and S(0 ; –16) MS = $\sqrt{180} = 6\sqrt{5}$ and QS = $\sqrt{80} = 4\sqrt{5}$ $\frac{LS}{RS} = \frac{QS}{MS} = \frac{2}{3}$ [prop theorem; RM ∥ LQ] **OR** [line ∥ one side of Δ/*lyn ∥ een sy v Δ*] Answer only: Full marks	✓ PS = 10 units ✓ TS = 15 units ✓ answer (3) ✓ MS = $6\sqrt{5}$ units ✓ QS = $4\sqrt{5}$ units ✓ answer (3)
8.7	area of PTMQ = area of ΔTSM – area of ΔPSQ $= \frac{1}{2}ST.\perp h_M - \frac{1}{2}PS.\perp h_Q$ $= \frac{1}{2}(15)(6) - \frac{1}{2}(10)(4)$ $= 45 - 20$ = 25 square units	✓ area of ΔTSM – area of ΔPSQ ✓ area ΔTSM = 45 ✓ area ΔPSQ = 20 ✓ answer (4)

QUESTION 9



9.1	$m_{TQ} = \frac{4-0}{0-3}$ $= -\frac{4}{3}$	✓ answer (1)
9.2	$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $RQ = \sqrt{(10-3)^2+(7-0)^2}$ $RQ = \sqrt{98} = 7\sqrt{2}$	✓substitution/*substitusie* ✓ answer in surd form (2)
9.3	$m_{FQ} = m_{TQ}$ $\frac{-8}{k-3} = -\frac{4}{3}$ $4k-12=24$ $k=9$ **OR/*OF*** $m_{FT} = m_{QT}$ $\frac{-8-4}{k-0} = -\frac{4}{3}$ $-36=-4k$ $k=9$	✓ equating gradients/*stel gradient gelyk* ✓ $m_{FQ} = \frac{-8}{k-3}$ ✓ simplification/*vereenvoudig* ✓ answer (4)
	OR/*OF* Equation of TQ: $y = -\frac{4}{3}x+4$ $-8 = -\frac{4}{3}k+4$ $k=9$	✓ gradient ✓ equation of TQ/*vgl van TQ* ✓ substitution of (k ; - 8) /*substitusie van* (k ; - 8) ✓ answer (4)

9.4	Using transformation/*Gebruik transformasie*: $\therefore S(7\ ;\ 11)$ **OR/*OF*** Midpoint of TR = midpoint of SQ [diag ||m/*hkle*||*m*] Midpoint of TR $= (5\ ;\ \frac{11}{2})$ $\frac{x_S+3}{2}=5$ and $\frac{y_S+0}{2}=\frac{11}{2}$ $\therefore x_s=7$ and $y_s=11$ $\therefore S(7\ ;\ 11)$ **OR/*OF***	✓✓ *x*–value/*waarde* ✓✓ *y*–value/*waarde* (4) ✓ *x*–value/*waarde* of/*van* T ✓ *y*–value/*waarde* of/*van* T ✓ *x*–value/*waarde* of/*van* S ✓ *y*–value/*waarde* of/*van* S (4)
9.5	$T\hat{S}R = T\hat{Q}R$ [opp ∠s of ||m/*teenoorst* ∠*e* ||*m*] $T\hat{Q}R = \alpha - \beta$ $\tan\alpha = m_{TQ} = -\frac{4}{3}$ $\therefore \alpha = 180° - 53{,}13° = 126{,}87°$ $\tan\beta = m_{RQ} = \frac{7}{7} = 1$ $\therefore \beta = 45°$ $T\hat{Q}R = 126{,}87° - 45°$ $= 81{,}87°$ $T\hat{S}R = 81{,}87°$	✓ $T\hat{Q}R = \alpha - \beta$ ✓ $\tan\alpha = m_{TQ}$ ✓ α ✓ $\tan\beta = m_{RQ}$ ✓ β ✓ answer (6)
9.6.1	$MQ = \sqrt{(5-3)^2+(2-0)^2}$ $MQ = \sqrt{8}$ $\frac{MQ}{RQ} = \frac{\sqrt{8}}{\sqrt{98}}$ $= \frac{2}{7}$ or 0,29 Answer only: full marks	✓substitution/*substitusie* ✓$MQ = \sqrt{8} = 2\sqrt{2}$ ✓ answer (3)
9.6.2	$\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{\frac{1}{2}.QM.\perp h}{\frac{1}{2}.QR.\perp h}$ [⊥h same/*dieselfde*] $= \frac{QM}{QR} = \frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm RQTS}} = \frac{\text{area of } \Delta TQM}{2 \times \text{area of } \Delta TQR}$ $= \frac{1}{2}\left(\frac{2}{7}\right) = \frac{1}{7}$	✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{2}{7}$ ✓ area parm RQTS = 2area ΔTQR ✓ answer (3)

QUESTION 10

10.1	$m_{AC}=\dfrac{1-(-4)}{7-(-3)}$ *OR* $\dfrac{-4-1}{-3-7}$ $=\dfrac{5}{10}=\dfrac{1}{2}$	✓ substitution ✓answer (2)
10.2.1	$y=\frac{1}{2}x+c$ $1=\frac{1}{2}(7)+c$ $c=-\frac{5}{2}$ $y=\frac{1}{2}x-2\frac{1}{2}$ **OR/*OF*** $y-y_1=\frac{1}{2}(x-x_1)$ $y-1=\frac{1}{2}(x-7)$ $y-1=\frac{1}{2}x-\frac{7}{2}$ $y=\frac{1}{2}x-2\frac{1}{2}$	✓ substitution M and A(7 ; 1) ✓equation (2)
10.2.2	$M\left(\frac{-2+8}{2};\frac{9+(-11)}{2}\right)$ $\therefore M(3;-1)$ Equation of AC: $y=\frac{1}{2}x-2\frac{1}{2}$ $y=\frac{1}{2}(3)-2\frac{1}{2}$ $y=-1$ **OR/OF** $y=\frac{1}{2}x-2\frac{1}{2}$ $-1=\frac{1}{2}x-2\frac{1}{2}$ $x=3$ $\therefore$ M lies on AC	✓x coordinate ✓y coordinate ✓ substitution of x ✓ conclusion (4)
	OR/OF	
	$M\left(\frac{-2+8}{2};\frac{9+(-11)}{2}\right)$ $\therefore M(3;-1)$ $m_{CM}=\frac{-4+1}{-3-3}=\frac{1}{2}$ $\therefore$ $m_{CM}=m_{AC}$ and C a common point $\therefore$ M lies on AC	✓ x coordinate ✓ y coordinate ✓ gradient of CM ✓ reasoning & conclusion (4)
10.3	$m_{BD}=\dfrac{9-(-11)}{-2-8}$ OR $\dfrac{(-11)-9}{8-(-2)}$ $=-2$ $m_{BD}\times m_{AC}=\frac{1}{2}\times-2$ $=-1$ $\therefore BD\perp AC$	✓ correct substitution ✓ m_{BD} ✓ product of gradients = –1 (3)

10.4.1	$\tan\theta = m_{BD} = -2$ $\therefore \theta = 116{,}57°$	✓ $\tan\theta = m_{BD}$ ✓ answer (2)
10.4.2	$\tan\beta = m_{BC}$ $m_{BC} = \dfrac{9-(-4)}{-2-(-3)}$ *OR* $\dfrac{-4-9}{-3-(-2)}$ $= 13$ $\beta = 85{,}6°$ $\therefore C\hat{B}D = 116{,}57° - 85{,}60°$ [ext $\angle$ of Δ] $= 30{,}97°$	✓ $m_{BC} = 13$ ✓ value of β ✓ answer (3)
	OR/OF $BD = \sqrt{500}$; $BC = \sqrt{170}$ & $CD = \sqrt{170}$ $CD^2 = BD^2 + BC^2 - 2BD.BC.\cos C\hat{B}D$ $170 = 500 + 170 - 2\sqrt{500}.\sqrt{170}.\cos C\hat{B}D$ $\cos C\hat{B}D = \dfrac{\sqrt{500}}{2\sqrt{170}} = 0{,}85749...$ $C\hat{B}D = 30{,}96°$	✓ subst into cos rule ✓ value of $\cos C\hat{B}D$ ✓ answer (3)
10.4.3	$AC = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ $= \sqrt{(7-(-3))^2+(1-(-4))^2}$ *OR* $\sqrt{((-3)-7)^2+((-4)-1)^2}$ $= \sqrt{100+25}$ $= \sqrt{125} = 5\sqrt{5} = 11{,}58$	✓ correct substitution into distance formula ✓ answer (2)
10.4.4	$BM = \sqrt{((-2)-3)^2+(9-(-1)^2}$ *OR* $\sqrt{(3-(-2))^2+((-1)-9)^2}$ $= \sqrt{125} = 5\sqrt{5}$ Area of $\Delta ABC = \dfrac{1}{2}\text{base} \times \perp \text{height}$ $= \dfrac{1}{2}(\sqrt{125})(\sqrt{125})$ $= 62{,}5$ square units Area of ABCD $= 2 \times 62{,}5$ $= 125$ square units	✓ correct substitution into distance formula ✓ BM ✓ substitution into area formula ✓ 62,5 ✓ 2 × ΔABC (5)
		[23]

PART B

QUESTION 5

1.1.1	$m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{KN} = \frac{2-(-1)}{-1-1}$ $= -\frac{3}{2}$ Answer only: Full marks	✓ correct substitution ✓ answer (2)
1.1.2	$\tan\theta = m_{KN} = -\frac{3}{2}$ $\theta = 180° - 56{,}31°$ $\theta = 123{,}69°$ Answer only: Full marks	✓ $\tan\theta = m_{KN} = -\frac{3}{2}$ ✓ answer (2)
1.2	Inclination KL = $123{,}69° - 78{,}69° = 45°$ [ext $\angle\Delta$] $\tan 45° = m_{KL} = 1$	✓ S ✓ $\tan 45° = m_{KL} = 1$ (2)
1.3	$y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$	✓ substitute (– 1 ; 2) and *m* ✓ equation (2)
	OR/*OF* $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y = x + 3$	✓ substitute (– 1 ; 2) and *m* ✓ equation (2)

1.4	$KN=\sqrt{(1+1)^2+(-1-2)^2}$ $KN=\sqrt{13}$ or 3,61 **Answer only: Full marks**	✓ substitute K and N into distance formula ✓ answer (2)
1.5.1	$(x+3)^2+(y+5)^2=13$ ...(1) L is a point on KL $y=x+3$ ...(2) (2) in (1): $(x+3)^2+(x+3+5)^2=13$ $x^2+6x+9+x^2+16x+64=13$ $2x^2+22x+60=0$ $x^2+11x+30=0$ $(x+5)(x+6)=0$ $x=-5$ or $x=-6$ $y=-2$ or $y=-3$ L$(-5\,;-2)$ or $(-6\,;-3)$	✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ x-values ✓ y-values (5)
	OR/OF	
	$(x+3)^2+(y+5)^2=13$ ...(1) L is a point on KL $y=x+3 \quad \therefore x=y-3$ ...(2) (2) in (1): $(y-3+3)^2+(y+5)^2=13$ $y^2+y^2+10y+25=13$ $2y^2+10y+12=0$ $y^2+5y+6=0$ $(y+2)(y+3)=0$ $y=-2$ or $y=-3$ $x=-5$ or $x=-6$ L$(-5\,;-2)$ or $(-6\,;-3)$	✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ y-values (both) ✓ x-values (both) (5)
1.5.2	Midpoint of KM: $(-2\,;-1{,}5)$ $\therefore \frac{x_L+1}{2}=-2$ and $\frac{y_L-1}{2}=-\frac{3}{2}$ $\therefore$ L$(-5\,;-2)$	✓ midpoint of KM ✓ x value ✓ y value (3)
	OR/*OF*	
	$m_{KN}=m_{LM}$ $\frac{y-(-5)}{x-(-3)}=-\frac{3}{2}$ $2(x+3+5)=-3(x+3)$ $2x+16=-3x-9$ $5x=-25$ $x=-5$ $\therefore$ L$(-5\,;-2)$ **Answer only: Full marks**	✓ $m_{LM}=m_{KN}$ ✓ x value ✓ y value (3)

	OR/*OF* N→M: $(x\,;y)\to(x-4\,;y-4)$ $\therefore$ L$(-1-4\,;2-4)$ **OR/OF** $\therefore$ L$(-5\,;-2)$ N→K: $(x\,;y)\to(x-2\,;y+3)$ $\therefore$ L$(-3-2\,;-5+3)$ $\therefore$ L$(-5\,;-2)$	
	OR/*OF* N→M: $(x\,;y)\to(x-4\,;y-4)$ $\therefore$ L$(-1-4\,;2-4)$ **OR/OF** $\therefore$ L$(-5\,;-2)$ N→K: $(x\,;y)\to(x-2\,;y+3)$ $\therefore$ L$(-3-2\,;-5+3)$ $\therefore$ L$(-5\,;-2)$	
	OR/*OF* N→M: $(x\,;y)\to(x-4\,;y-4)$ $\therefore$ L$(-1-4\,;2-4)$ **OR/OF** $\therefore$ L$(-5\,;-2)$ N→K: $(x\,;y)\to(x-2\,;y+3)$ $\therefore$ L$(-3-2\,;-5+3)$ $\therefore$ L$(-5\,;-2)$	
	OR/*OF* N→M: $(x\,;y)\to(x-4\,;y-4)$ $\therefore$ L$(-1-4\,;2-4)$ **OR/OF** $\therefore$ L$(-5\,;-2)$ N→K: $(x\,;y)\to(x-2\,;y+3)$ $\therefore$ L$(-3-2\,;-5+3)$ $\therefore$ L$(-5\,;-2)$	
	OR/*OF* N→M: $(x\,;y)\to(x-4\,;y-4)$ $\therefore$ L$(-1-4\,;2-4)$ **OR/OF** $\therefore$ L$(-5\,;-2)$ N→K: $(x\,;y)\to(x-2\,;y+3)$ $\therefore$ L$(-3-2\,;-5+3)$ $\therefore$ L$(-5\,;-2)$	✓ transformation ✓x value ✓ y value (3)
1.6	T(–6 ; –3) (from Question 3.5.1) $KT=\sqrt{(-1-(-6))^2+(2-(-3))^2}$ $=\sqrt{50}$ $KN=\sqrt{13}$ (CA from 3.4) Area of $\Delta KTN=\frac{1}{2}KT.KN\sin L\hat{K}N$ $=\frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78{,}69°$ = 12,50 square units	✓ coordinates of T ✓ length of KT ✓ substitution into area rule ✓ answer (4)

	OR/*OF*	
	In ΔKLM:	
	$\dfrac{TL}{\sin 22{,}62°} = \dfrac{\sqrt{13}}{\sin 78{,}69°}$	
	$TL = 1{,}414..$	✓ length of TL
	$KL = \sqrt{(-1-(-5))^2 + (2-(-2))^2}$	
	$= \sqrt{32}$	
	$\therefore\ KT = 7{,}0708\ldots$	✓ length of KT
	Area of $\Delta KTN = \dfrac{1}{2} KT.KN \sin L\hat{K}N$	
	$= \dfrac{1}{2}(7{,}0708).\sqrt{13} \sin 78{,}69°$	✓ substitution into area rule
	$= 12{,}50$ square units	✓ answer (4)
		[22]

QUESTION 6

6.1	$x^2+y^2+8x+4y-38=0$ $x^2+8x+16+y^2+4y+4=16+4+38$ $(x+4)^2+(y+2)^2=58$ Centre is $(-4\ ;-2)$ and the radius is $\sqrt{58}$	✓completing the square (both or one) ✓ factor form ✓ centre ✓ radius (4)
6.2	Centre of second circle is $(4\ ;6)$ Distance between centres is $\sqrt{(4+4)^2+(6+2)}=\sqrt{128}=11{,}31$	✓centre ✓distance (2)
6.3	Sum of radii = $\sqrt{58}+\sqrt{26}=12{,}71$ Distance between centres is 11,31. sum of the radii > distance between the centres $\therefore$ the circles must overlap and hence the circles must intersect.	✓✓ sum of radii ✓ conclusion (3)
6.4	Equation of second circle: $(x-4)^2+(y-6)^2=26$ $x^2-8x+16+y^2-12y+36=26$ $x^2-8x+y^2-12y+26=0$ Let $(x\ ;y)$ be either of the two points on intersection. Then $x^2+y^2+8x+4y-38=0$ and $x^2+y^2-8x-12y+26=0$ Subtract $16y+16x-64=0$ $y=-x+4$ Both points of intersection lie on this line. $\therefore$ $y=-x+4$ is the equation of the common chord. **OR**	✓ equation of circle in form = 0 ✓ statement – two points of intersection ✓ subtracting ✓ simplification (4)

	Check that the line $y = -x + 4$ cuts the two circles at the same points:	
	$(x-4)^2 + (-x-2)^2 = 26$ $x^2 - 8x + 16 + x^2 + 4x + 4 = 26$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$	✓ substitution ✓ answer
	$x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$ $x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $x = 3$ or $x = -1$	✓ substitution ✓ answer (4) **[13]**

QUESTION 7

7.1	M(8 ; – 4)	✓ coordinates (1)
7.2	$OM = \sqrt{(8-0)^2+(-4-0)^2}$ $= \sqrt{80}$ or $4\sqrt{5}$ units	✓ substitution into distance formula ✓ $\sqrt{80}$ *or* $4\sqrt{5}$ (2)
7.3	$ON = OM - NM$ $= \sqrt{80} - \sqrt{45}$ $= 4\sqrt{5} - 3\sqrt{5}$ $= \sqrt{5}$ *units*	✓ $ON = OM - NM$ ✓ length of NM ✓ answer (3)
7.4	$M\hat{T}P = 90°$ (tangent/*raaklyn* ⊥ radius) $\therefore\ O\hat{M}T = 90°$ (alternate ∠'s /*verwissellende* ∠'*e* ; TP | | OM)	✓ Statement + reason ✓ answer (2)
7.5	$m_{MT} \cdot m_{OM} = -1$ $m_{OM} = \frac{-4-0}{8-0} = -\frac{1}{2}$ $m_{MT} = 2$ $y+4 = 2(x-8)$ $y = 2x-20$ **OR** $y = 2x + c$ $-4 = 2(8) + c$ $c = -20$ $y = 2x - 20$	✓ ✓ m_{OM} ✓ m_{MT} ✓ substitution of m and (8 ; –4) ✓ equation MT (5)
7.6	$(x-8)^2+(y+4)^2 = 45$ $(x-8)^2+(2x-20+4)^2 = 45$ $(x-8)^2+(2x-16)^2 = 45$ $x^2-16x+64+4x^2-64x+256-45=0$ $5x^2-80x+275=0$ $x^2-16x+55=0$ $(x-11)(x-5)=0$ $x = 11$ $y = 2(11)-20$ $y = 2$ $\therefore$ T(11 ; 2)	✓ substitution ✓ expansion ✓ standard form ✓ factors ✓ $x = 11$ ✓ substitution (6) **[19]**

QUESTION 8

8.1	line from centre to **midpt of chord** / *lyn vanaf midpt na midpt van koord*	✓ answer (1)
8.2	$m_{ST} = \frac{8-5}{-3-0}$ $= -1$ $m_{ST} \times m_{NP} = -1$ [TS ⊥ NP] $\therefore m_{NP} = 1$ $\therefore y = x + c$ $8 = -3 + c$ $c = 11$ $\therefore y = x + 11$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 8 = 1(x + 3)$ $y = x + 11$	✓ subst (–3 ; 8) and (0 ; 5) into gradient formula ✓ m_{ST} ✓ m_{NP} ✓ subst (–3 ; 8) into equation of a line ✓ equation (5)
8.3	P(0 ; 11) [y-intercept of chord NP] ∴ radius is 6 units R(0 ; –1) Equations of the tangents to the circle parallel to the x-axis/ *Vgls van die raaklyne aan die sirkel || aan die x-as*: $y = 11$ and $y = -1$	✓ coordinates of P/ koördinate v P ✓ coordinates of R koördinate van R ✓✓ answers (4)
8.4	M(–11 ; 0) [x–intercept of/*x-afsnit van* NP] $MT = \sqrt{(0-11)^2 + (5-0)^2}$ $MT = \sqrt{146} = 12{,}08$	✓✓ coordinates of M ✓ substitution ✓ answer (4)

8.5	MT = diameter/*middellyn* [conv∠ in $\frac{1}{2}$ circle/*omgek ∠in $\frac{1}{2}$ sirkel*] radius $= \frac{\sqrt{146}}{2}$ units Centre of circle/*Middelpunt v sirkel* = Midpoint MT /*Middelpunt MT* $= \left(\frac{-11}{2} ; \frac{5}{2}\right)$ Equation of circle through S, T and M: $\left(x + \frac{11}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{146}{4}$ **OR/*OF*** $\left(x + 5\frac{1}{2}\right)^2 + \left(y - 2\frac{1}{2}\right)^2 = \frac{73}{2} = 6{,}04$	✓ radius of circle ✓ x value of M ✓ y value of M ✓ LHS of equation ✓ RHS of equation (5) **[19]**

QUESTION 9

9.1	$r = \text{MN} = 5$	✓ answer/*antw* (1)
9.2	$(x-5)^2 + (y-4)^2 = 25$	✓equation/*vgl* (1)
9.3	A(x ; 0) $(x-5)^2 + (0-4)^2 = 25$ $x^2 - 10x + 25 + 16 = 25$ $x^2 - 10x + 16 = 0$ $(x-8)(x-2) = 0$ $\therefore x = 8$ or/*of* $x = 2$ $\therefore$ A(2 ; 0) **OR/*OF*** $(x-5)^2 + (0-4)^2 = 25$ $(x-5)^2 + 16 = 25$ $(x-5)^2 = 9$ $(x-5) = \pm 3$ $\therefore x = 8$ or/*of* $x = 2$ $\therefore$ A(2 ; 0)	✓ substitute into eq/ *vervang in vgl* $y = 0$ ✓ standard form/ *standaardvorm* or perfect square form/*kwadr vorm* ✓ answer/*antw* (3)
9.4.1	$m_{\text{MB}} = \dfrac{4-0}{5-8}$ $= -\dfrac{4}{3}$	✓ subst M and B into form/*vervang M and B in form* ✓ $m_{\text{MB}} = -\dfrac{4}{3}$ (2)

9.4.2	$m_{MB} \times m_{PB} = -1$ (tangent $\perp$ radius/ *rkl* $\perp$ *radius*) $m_{PB} = \frac{3}{4}$ $y = \frac{3}{4}x + c$ **OR/*OF*** $y - y_1 = \frac{3}{4}(x - x_1)$ $0 = \frac{3}{4}(8) + c$ $y - 0 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 6$ $y = \frac{3}{4}x - 6$	✓ $m_{MB} \times m_{PB} = -1$ ✓ $m_{PB} = \frac{3}{4}$ ✓ equation/*vgl* (3)
9.5	$y_K = y_M + r = 4 + 5$ $y = 9$	✓ 9 ✓ equation/*vgl* (2)
9.6	At/*By* L: $\frac{3}{4}x - 6 = 9$ $3x - 24 = 36$ $3x = 60$ $x = 20$ $\therefore$ L(20 ; 9)	✓ equating simultaneously ✓ simplification (2)
9.7	L(20 ; 9) $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **OR/OF** $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(20-5)^2 + (9-4)^2}$ $= \sqrt{(15)^2 + (5)^2}$ $= \sqrt{225 + 25}$ $= \sqrt{(5)^2(9+1)}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$	✓ correct subst into distance formula/ *korrekte subst in afstand-formule* ✓ answer in surd form/*antw in wortelvorm* (2)
9.8	**MK ⊥ KL OR/OF** $\mathbf{M\hat{K}L} = 90°$ (radius $\perp$ tangent/*radius* $\perp$ *rkl*) $\therefore$ML is a diameter as it subtends a right angle/*ML is middellyn* $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore (x - 12{,}5)^2 + (y - 6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$ **OR/*OF***	✓ S ✓ value of/*waarde van r* ✓ $x = 12{,}5$ ✓ $y = 6{,}5$ ✓ answer in correct form/ *antw in korrekte vorm* (5)

	MK $\perp$ KL OR/OF $\mathbf{M\hat{K}L} = 90°$ (radius $\perp$ tangent/*radius $\perp$ rkl*) $\therefore$ML is a diameter as it subtends a right angle/*ML is middellyn* Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12,5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6,5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $(x-12,5)^2 + (y-6,5)^2 = r^2$ subst (5 ; 4): $(5-12,5)^2 + (4-6,5)^2 = r^2$ $62,5 = r^2$ $\therefore (x-12,5)^2 + (y-6,5)^2 = \frac{250}{4} = \frac{125}{2} = 62,5$ **OR/*OF*** By symmetry about LM/*deur simmetrie om LM*: **MK $\perp$ KL OR/OF** $\mathbf{M\hat{K}L} = 90°$ (radius $\perp$ tangent/*radius $\perp$ rkl*) $\therefore$ML is a diameter as it subtends a right angle/*ML is middellyn* ML is a diameter /*ML is 'n middellyn* $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or /of 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12,5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6,5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore (x-12,5)^2 + (y-6,5)^2 = \frac{250}{4} = \frac{125}{2} = 62,5$	✓ S ✓ $x = 12,5$ ✓ $y = 6,5$ ✓ value of/*waarde van* r^2 ✓ answer in correct form/*antw in korrekte vorm* (5) ✓ S ✓ value of/*waarde van r* ✓ $x = 12,5$ ✓ $y = 6,5$ ✓ answer in correct form/*antw in korrekte vorm* (5) **[21]**

QUESTION 10

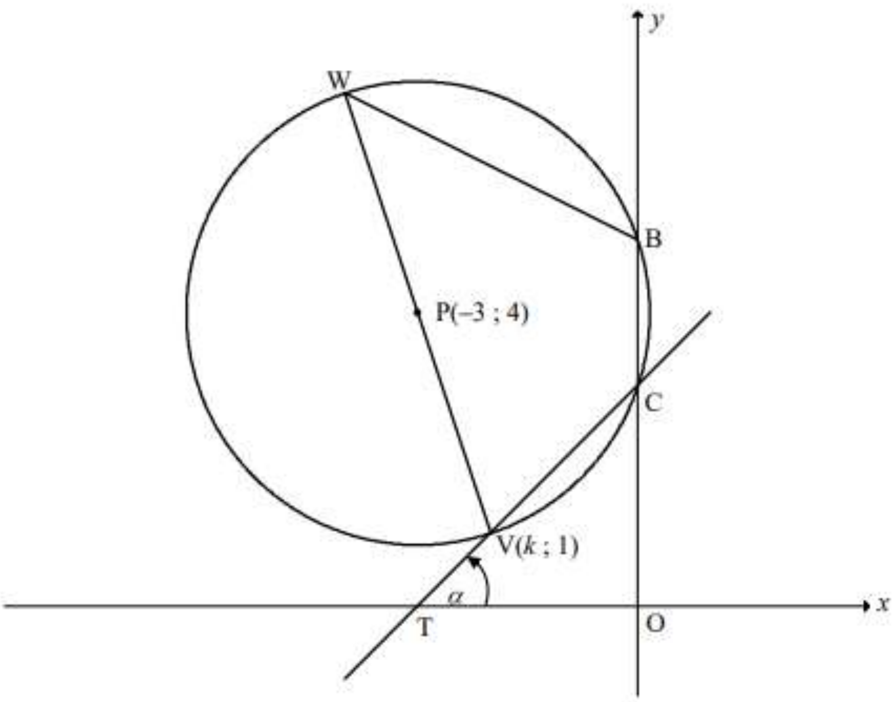
10.1	$\angle$ in semi circle/ $\angle$ at centre = 2$\angle$ on circle $\angle$ *in halfsirkel* /$\angle$ by middelpt = 2$\angle$ op sirkel	✓ R (1)
10.2	$m_{TS} = \frac{7-2}{3-5}$ $= -\frac{5}{2}$	✓ substitution ✓ m_{TS} (2)
10.3	$m_{TS} \times m_{RS} = -1$ [TS$\perp$SR] $\therefore m_{RS} = \frac{2}{5}$ $y = \frac{2}{5}x + c$ $2 = \frac{2}{5}(5) + c$ $c = 0$ $y = \frac{2}{5}x$ **OR/*OF***	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)

	$m_{TS} \times m_{RS} = -1$ [TS ⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)
10.4.1	$r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ **OR/OF** $TM = \sqrt{(3-9)^2 + \left(7 - 6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ r ✓ answer (2) ✓ substitution ✓ answer (2)
10.4.2	$M\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9$ and $\frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore R(15 ; 6)$ Answer only: full marks Answer only: only 1 coordinate correct (1 mark) **OR/*OF*** $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore R\left(9 + 6 ; 6\frac{1}{2} - \frac{1}{2}\right) = R(15 ; 6)$ **OR/OF**	✓ M ✓ x coordinate ✓ y coordinate (3) ✓ M ✓ x coordinate ✓ y coordinate (3)

	$m_{TM}=\dfrac{9-3}{6\frac{1}{2}-7}=-\dfrac{1}{12}$ $TM:\quad 7=-\dfrac{1}{12}(3)+c \quad y=-\dfrac{1}{12}x+\dfrac{29}{4}$ ....(1) $SR:\quad y=\dfrac{2}{5}x$ ....(2) $\dfrac{2}{5}x=-\dfrac{1}{12}x+\dfrac{29}{4}$ $\dfrac{29}{60}x=\dfrac{29}{4}$ $\therefore x=15$ $\therefore y=\dfrac{2}{5}(15)=6$	✓ equating ✓ x coordinate ✓ y coordinate (3)
10.4.3	$\text{ST}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\text{ST}=\sqrt{(5-3)^2+(2-7)^2}$ $\text{ST}=\sqrt{4+25}=\sqrt{29}$ $\sin \text{R}=\dfrac{\text{TS}}{\text{TR}}=\dfrac{\sqrt{29}}{\sqrt{145}}\ or\ \dfrac{\sqrt{5}}{5}\ or\ \dfrac{1}{\sqrt{5}}\ or\ 0{,}45$ **OR/OF** $TS=\sqrt{29}$ $SR=2\sqrt{29}$ area of $\Delta\text{TSR}=\dfrac{1}{2}\left(\sqrt{29}\right)\left(2\sqrt{29}\right)=29$ $29=\dfrac{1}{2}(\sqrt{145})(2\sqrt{29})\sin R$ $\sin R=\dfrac{\sqrt{5}}{5}\ or\ \dfrac{1}{\sqrt{5}}$	✓substitution ✓ answer ✓ ratio (3) ✓area ✓ rule ✓ ratio (3)
10.4.4	$m_{\text{TR}}=\dfrac{7-6\frac{1}{2}}{3-9}=-\dfrac{1}{12}$ **OR/*OF*** $m_{\text{TR}}=\dfrac{7-6}{3-15}=-\dfrac{1}{12}$ $m_{\text{TR}}\times m_{\text{KTL}}=-1$ [$r\perp$ tangent] $m_{\text{KTL}}=12$ $y-y_1=12(x-x_1)$ $y-7=12(x-3)$ $y=12x-29$ substitute $\text{K}(a;b)$: $b=12a-29$ **OR/*OF***	✓ $m_{\text{TR}}=-\dfrac{1}{12}$ ✓ $m_{\text{KTL}}=12$ ✓ $y=12x-29$ (3)

	$m_{TR} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ $m_{TR} \times m_{KTL} = -1$ [$r \perp$ tangent] $\dfrac{b-7}{a-3} = 12$ $b-7 = 12(a-3)$ $b = 12a-29$	✓ $m_{TR} = -\dfrac{1}{12}$ ✓ $m_{KTL} = 12$ ✓substitution (3 ; 7) & (a ; b) (3)
	OR/*OF* $KR^2 = TR^2 + TK^2$ $(a-15)^2 + (b-6)^2 = (15-3)^2 + (6-7)^2 + (a-3)^2 + (b-7)^2$ $-30a + 225 - 12b + 36 = 144 + 1 - 6a + 9 - 14b + 49$ $2b = 24a - 58$ $b = 12a - 29$	✓subst into Pyth ✓multiplication ✓simplification (3)
10.4.5	$TK = TR$ $\sqrt{(a-3)^2 + (b-7)^2} = \sqrt{145}$ $(a-3)^2 + (b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2 + (12a-29-7)^2 = 145$ $(a-3)^2 + (12a-36)^2 = 145$ $a^2 - 6a + 9 + 144a^2 - 864a + 1296 - 145 = 0$ $145a^2 - 870a + 1160 = 0$ $a = \dfrac{870 \pm \sqrt{(870)^2 - 4(145)(1160)}}{290}$ $a = 2$ or $a = 4$ $\therefore b = 12(2) - 29 = -5$ or $b = 12(4) - 29 = 19$ $\therefore K(2 ; -5)$ **OR/*OF***	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓standard form ✓ subst into formula or factorise ✓ values of a ✓ value of b (6)

	TK = TR $\sqrt{(a-3)^2+(b-7)^2}=\sqrt{145}$ $(a-3)^2+(b-7)^2=145$ Substitute $b=12a-29$ [from 4.4.4] $(a-3)^2+(12a-29-7)^2=145$ $(a-3)^2+(12a-36)^2=145$ $(a-3)^2+144(a-3)^2=145$ $(a-3)^2=1$ $a-3=\pm1$ $a=2$ or 4 $\therefore b=12(2)-29$ or $b=12(4)-29$ $=-5$ $=19$ $\therefore$ K$(2;-5)$	✓ substitution into distance formula ✓ substitution of $b=12a-29$ ✓ $(a-3)^2=1$ ✓ ±1 ✓ values of a ✓ value of b (6)
	OR/*OF* $KR^2=TR^2+TK^2$ $(a-15)^2+(b-6)^2=145+145$ $(a-15)^2+(12a-29-6)^2=290$ $(a-15)^2+(12a-35)^2=290$ $a^2-30a+225+144a^2-840a+1225=290$ $145a^2-870a+1160=0$ $a^2-6a+8=0$ $\therefore(a-2)(a-4)=0$ $a=2$ or $a=4$ $\therefore b=12(2)-29$ or $b=12(4)-29$ $=-5$ $=19$ K$(2;-5)$	✓ substitution ✓ substitution of $b=12a-29$ ✓standard form ✓ factors ✓ values of a ✓ value of b (6)
		[23]

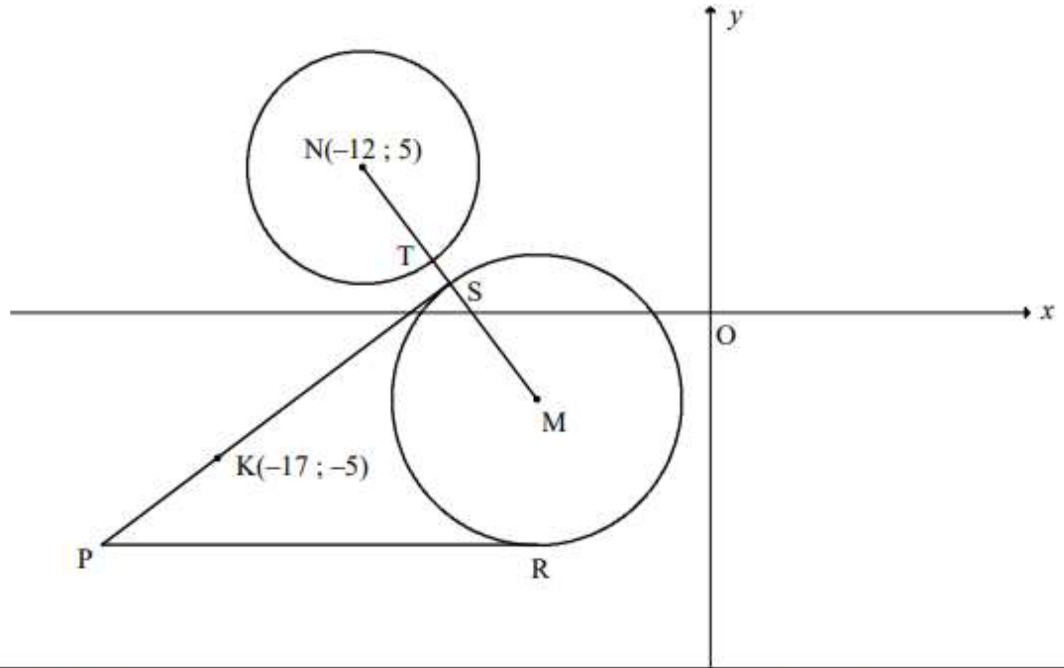


4.1	$PV = r = \sqrt{10}$ $PV = \sqrt{(k-(-3))^2 + (1-4)^2} = \sqrt{10}$ $(PV)^2 = (k-(-3))^2 + (1-4)^2 = 10$ $k^2 + 6k + 9 + 9 = 10$ **OR** $(k+3)^2 + 9 = 10$ $k^2 + 6k + 8 = 0$ $(k+3)^2 = 1$ $(k+4)(k+2) = 0$ $k+3 = 1$ or $k+3 = -1$ $k = -4$ or $k = -2$ $\therefore k = -2$	✓ $PV = r = \sqrt{10}$ ✓ substitution into distance formula ✓ standard form ✓ factors ✓ answer (5)
4.2	$x^2 + 6x + y^2 - 8y + 15 = 0$ y-intercepts: $(0)^2 + 6(0) + y^2 - 8y + 15 = 0$ $(y-3)(y-5) = 0$ $y_C = 3$ or $y_B = 5$ $\therefore BC = 2$ units	✓ $x = 0$ ✓ factors ✓ both values ✓ answer (4)

4.3.1	$m_{TC}=\dfrac{3-1}{0-(-2)}$ $=1$ $\tan\alpha=1$ $\therefore \alpha=45°$ **OR** $y=mx+3$ $1=m(-2)+3$ $m_{TC}=1$ $\tan\alpha=1$ $\therefore \alpha=45°$	✓ substitution into gradient formula ✓ $\tan\alpha=1$ ✓ answer (3) ✓ substitution into equation of a line ✓ $\tan\alpha=1$ ✓ answer (3)
4.3.2	$B\hat{C}V=135°$ [ext ∠ of Δ/*buite* ∠ v Δ] $\therefore V\hat{W}B=45°$ [opp ∠s of cyclic quad/*teenoorst. ∠e v kvh*] Answer only: Full marks **OR** $T\hat{C}O=45°$ [∠s of Δ/∠*e* v Δ] $\therefore V\hat{W}B=45°$ [ext ∠s of cyclic quad/*buite* ∠ *v kvh*] Answer only: Full marks	✓ $B\hat{C}V=135°$ ✓ answer (2) ✓ $T\hat{C}O=45°$ ✓ answer (2)
4.4.1	$Q(-3;-2)$	✓ x_Q ✓ y_Q (2)
4.4.2	$(x+3)^2+(y+2)^2=10$	✓ LHS ✓ RHS (2)
4.4.3	$x=-2$ or $x=-4$	✓ $x=-2$ ✓ $x=-4$ (2)
		[20]

QUESTION 5

5.1.1

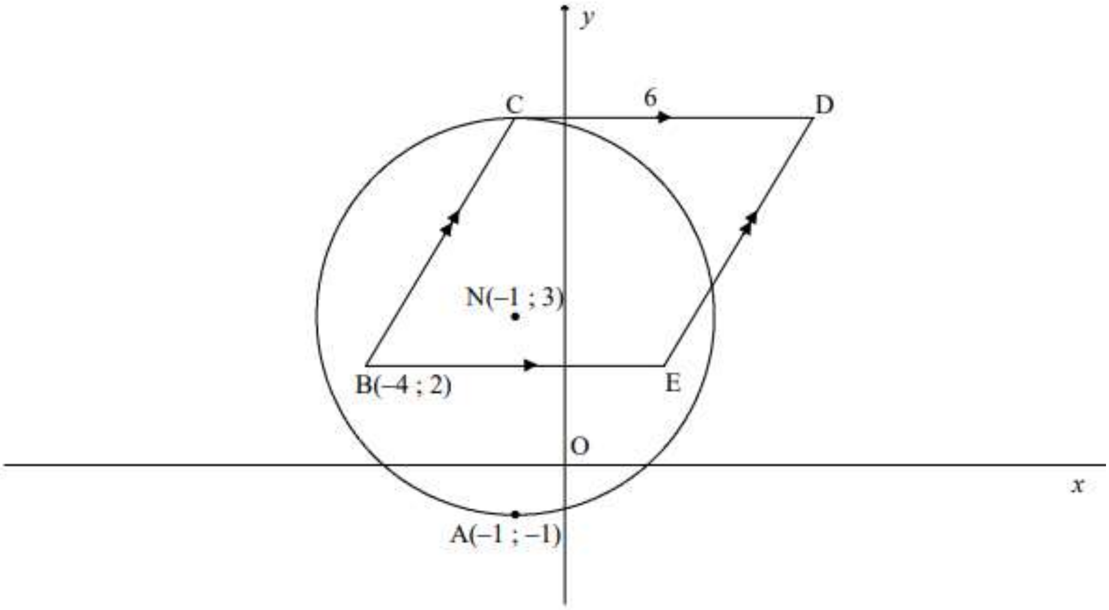


4.1	$M(-6;-3)$	✓ –6 ✓ –3 (2)
4.2.1	$x^2+y^2+24x-10y+153=0$ $(x+12)^2+(y-5)^2=-153+144+25$ $(x+12)^2+(y-5)^2=16$ $r^2=16$ $r=4$ units	✓ $r^2=-153+144+25$ ✓ length of radius (2)
4.2.2	$NM=\sqrt{(-12-(-6))^2+(5-(-3))^2}$ NM = 10 units SM = 5 units $\therefore$ TS = 10 – 5 – 4 = 1 unit	✓ substitution into distance formula ✓ NM = 10 units ✓ SM = 5 units ✓ answer (4)
4.3.1	R(–6 ; –8) $y=-8$	✓ $y_R=-8$ ✓ answer (2)

4.3.2	$m_{NM}=\frac{5-(-3)}{-12-(-6)}$ $m_{NM}=-\frac{4}{3}$ $m_{tangent}=\frac{3}{4}$ $-5=\frac{3}{4}(-17)+c$ **OR/*OF*** $y-y_1=\frac{3}{4}(x-x_1)$ $c=\frac{31}{4}$ $y-(-5)=\frac{3}{4}(x-(-17))$ $y=\frac{3}{4}x+\frac{31}{4}$ $y=\frac{3}{4}x+\frac{31}{4}$	✓ substitution ✓ $m_{NM}=-\frac{4}{3}$ ✓ $m_{tangent}=\frac{3}{4}$ ✓ substitution of m and N ✓ equation (5)

4.4.1	$-8=\frac{3}{4}x+\frac{31}{4}$ $-32=3x+31$ $3x=-63$ $x=-21$ P(–21 ; –8) R(–6 ; –8) PR = PS = 15 units [tangents from same point] MS = MR = 5 units Perimeter PSMR = 15 + 15 + 5 + 5 = 40 units	✓ $-8=\frac{3}{4}x+\frac{31}{4}$ ✓ $x=-21$ ✓ PR = PS = 15 units ✓ MS = MR = 5 units ✓ answer (5)

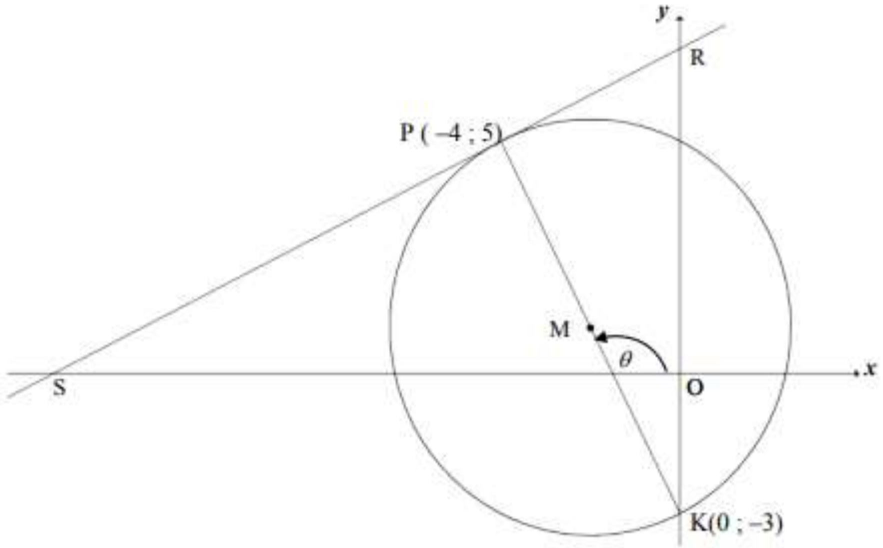
4.4.2	$\frac{\text{area of }\Delta\text{NPS}}{\text{area of quadrilateral PSMR}}$ $\frac{\frac{1}{2}\text{NS.SP}}{\frac{1}{2}\text{SP.MS}+\frac{1}{2}\text{MR.PR}}$ $=\frac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$ $=\frac{1}{2}$ **OR** ΔNPS ≡ ΔSPM ≡ ΔMPR $\frac{\text{area of }\Delta\text{NPS}}{\text{area of quadrilateral PSMR}}$ $=\frac{1}{2}$	✓ substitution ✓ answer (2) ✓ congruent ✓ answer (2)
		[22]



4.1	Radius = 4 units/*eenhede*	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1 ; 7)$	✓x value ✓ y value (2)
4.2.2	CD = 6 units $\therefore D(5 ; 7)$	✓x value ✓ y value (2)

4.2.3	$\perp h = 5$ units DC = 6 units Area $\Delta BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ **OR/*OF*** $\perp h = 5$ units DC = 6 units Area $\Delta BCD = \frac{1}{2}[\text{Area of } \|^{m}]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

4.3.1	M(3 ; –1) [reflection of N(–1 ; 3) about the line $y = x$] $\therefore \text{MN} = \sqrt{(3-(-1))^2 + (-1-3)^2}$ $\text{MN} = \sqrt{32} = 4\sqrt{2} = 5{,}66 \text{ units}$	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
4.3.2	M(3 ; –1) $m_{MN} = \frac{3-(-1)}{-1-3} = -1$ MN: $-1 = -(3) + c$ or $y - 3 = -1(x+1)$ $c = 2$ $y - 3 = -x - 1$ $\therefore y = -x + 2$ $y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint (1 ; 1)	✓ equation of MN ✓ equating AF & MN ✓x value ✓ y value (4)



4.1.1	$m_{PK} = \frac{5-(-3)}{-4-0}$ $= -2$ PK $\perp$ SR [radius $\perp$ tangent/*raaklyn*] $\therefore\ m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	✓ substitution P & K into gradient formula ✓ gradient of PK ✓ PK $\perp$ SR **OR** r $\perp$ tangent ✓ answer (4)

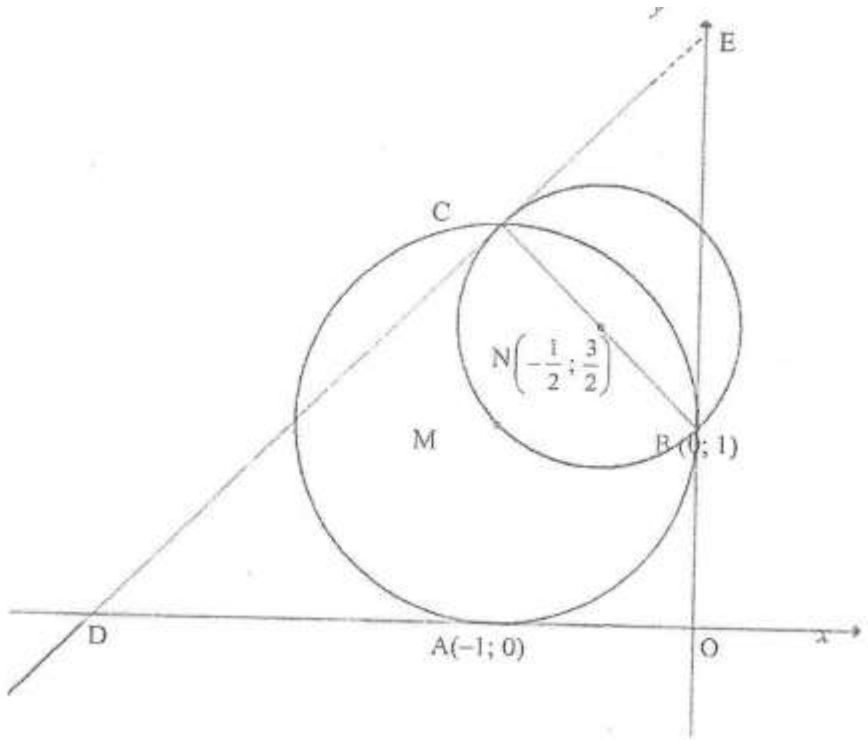
4.1.2	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c$ **OR/*OF*** $(y-5) = \frac{1}{2}(x-(-4))$ $c = 7$ $(y-5) = \frac{1}{2}x + 2$ $y = \frac{1}{2}x + 7$ $y = \frac{1}{2}x + 7$	✓ substitution of *m* and P ✓ equation (2)

4.1.3	$M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)$ $\therefore M(-2;1)$ $r^2=(x_2-x_1)^2+(y_2-y_1)^2$ $r^2=(-2+4)^2+(1-5)^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20$ *or* $(\sqrt{20})^2$ **OR/*OF*** $M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)$ $\therefore M(-2;1)$ $(x+2)^2+(y-1)^2=r^2$ $(-4+2)^2+(5-1)^2=r^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20$ *or* $(\sqrt{20})^2$ **OR/*OF*** $M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)$ $\therefore M(-2;1)$ $PK=\sqrt{(-4-0)^2+(5-(-3))^2}=\sqrt{80}$ $r=\frac{\sqrt{80}}{2}=\sqrt{20}$ $\therefore (x+2)^2+(y-1)^2=20$ *or* $(\sqrt{20})^2$	✓ x value of M ✓ y value of M ✓ $r^2=20$ ✓ equation (4) ✓✓ M $(-2;1)$ $r^2=20$ ✓ equation (4) ✓✓ M $(-2;1)$ $r^2=20$ ✓ equation (4)

4.1.4	$\tan\theta=m_{PK}=-2$ $\therefore \theta=180°-63{,}43°$ $=116{,}57°$ $P\hat{K}R=116{,}57°-90°$ [ext $\angle$of ΔMOK] $=26{,}57°$	✓ $\tan\theta=-2$ ✓ size of θ ✓ answer (3)

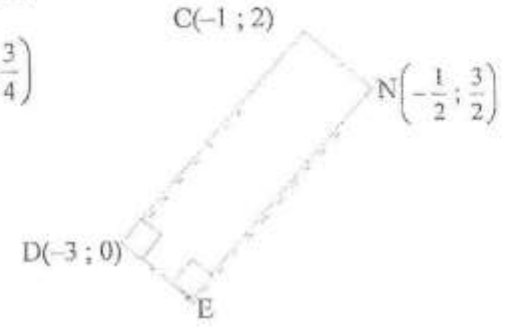
4.1.5	RS ‖ tangent at K(0 ; −3) $\therefore m_{PS} = m_{tang} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$ **OR/*OF*** $m_{PK} = \frac{1-5}{-2+4} = -2$ $m_{PK} \times m_{tan\,g} = -1$ [radius ⊥ tangent/*raaklyn*] $\therefore m_{tan\,g} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$	✓ gradient ✓ equation (2) ✓ gradient ✓ equation (2)
4.2	$t \in (-3\ ;\ 7)$ **OR/*OF*** $-3 < t < 7$	✓ −3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3) ✓ −3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3)
4.3	RS: $y = \frac{1}{2}x + 7$ $\therefore$ S(−14 ; 0) SP $= \sqrt{(-14-(-4))^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125}$ Area ΔSMK $= \frac{1}{2}$. MK . SP $= \frac{1}{2}(\sqrt{20})(\sqrt{125})$ = 25 square units	✓ coordinates of S ✓ length of SP ✓ correct base & height into Area rule ✓ correct substitution ✓ answer (5)

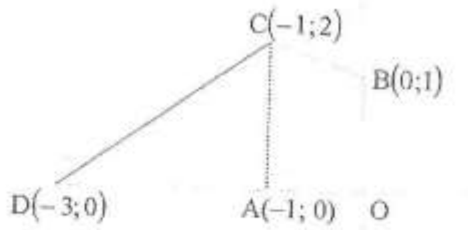
4.6	$RS = \sqrt{(-3+7,5)^2+(4-1)^2}$ OR/OF $RM = \sqrt{(-3+7,5)^2+(-2-1)^2}$ $RS = \frac{3\sqrt{13}}{2} = 5,41$ area of RSNM = 2area of ΔRSN $= 2\left(\frac{1}{2}\right)(\sqrt{13})\left(\frac{3\sqrt{13}}{2}\right)$ $= \frac{39}{2}$ **OR/*OF*** 19,5 square units **OR/*OF***	✓ RS **OR** RM ✓ method ✓ $\sqrt{13}$ and $\left(\frac{3\sqrt{13}}{2}\right)$ ✓ answer (4)

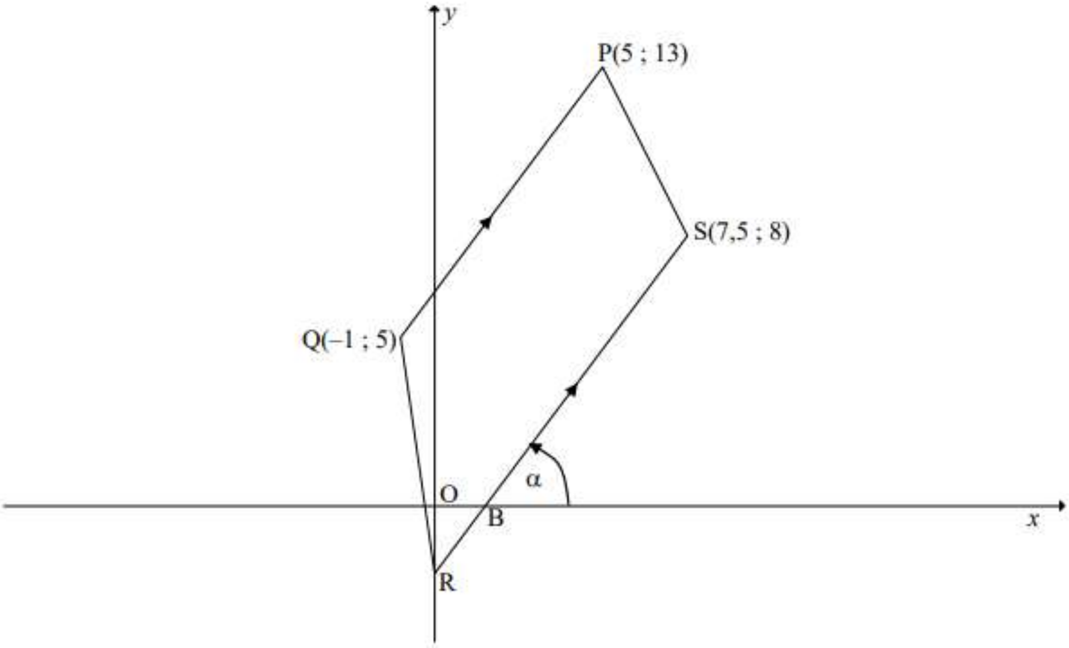


4.1	M(−1;1) $(x+1)^2+(y-1)^2=1$ Answer only: Full marks	✓M(−1;1) ✓LHS ✓ RHS (3)
4.2	Midpoint of CB, N: (– 0,5 ; 1,5)	

4.2	Midpoint of CB, N: (– 0,5 ; 1,5) $\therefore \frac{x_C+0}{2}=-\frac{1}{2}$ and $\frac{y_C+1}{2}=\frac{3}{2}$ Answer only: Full marks ∴ C(– 1 ; 2) **OR** B→N: $(x;y)\rightarrow(x-0{,}5;y+0{,}5)$ N→C: $(x;y)\rightarrow(x-0{,}5;y+0{,}5)$ Answer only: Full marks ∴ C(– 0,5 – 0,5 ; 1,5 + 0,5) ∴ C(– 1 ; 2)	(3) ✓x value ✓ y value (2) ✓x value ✓ y value (2)

4.3	$m_{\text{radius}} = \frac{2-1}{-1-0}$ OR $\frac{2-(-\frac{1}{2})}{-1-\frac{3}{2}}$ OR $\frac{0-(-\frac{1}{2})}{1-\frac{3}{2}}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y = mx + c$ $y = x + c$ $2 = 1(-1) + c$ $c = 3$ $\therefore y = x + 3$ $y - x = 3$	✓ m_{radius} ✓ m_{tangent} ✓ substitute $(-1\,;2)$ and m ✓ simplification (4)
	OR $m_{\text{radius}} = \frac{2-1}{-1-0}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y - y_1 = m(x - x_1)$ $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y - 2 = x + 1$ $\therefore y = x + 3$ $y - x = 3$	✓ m_{radius} ✓ m_{tangent} ✓ substitute $(-1\,;2)$ and m ✓ simplification (4)
4.4	Tangents to circle: $y = x + 3$ and $y = x + 1$ $\therefore t > 3$ or $t < 1$ Answers only: Full marks	✓ $y = x + 1$ ✓ $t > 3$ ✓ $t < 1$ (3)
4.5	Draw rectangle CNED: Midpt of DN $\left(-\frac{7}{4}\,;\frac{3}{4}\right)$ $\therefore E(-\frac{5}{2}\,;-\frac{1}{2})$	✓ midpt of DN ✓x value ✓ y value (3)
		
	OR/*OF* $D(-3\,;0)$ C→N: $(x\,;y) \rightarrow (x+0{,}5\,;y-0{,}5)$ D→E: $D(x\,;y) \rightarrow E(x+0{,}5\,;y-0{,}5)$ $\therefore E(-3+0{,}5\,;0-0{,}5)$ $\therefore E(-2{,}5\,;-0{,}5)$ Answer only: Full marks	✓ coordinates of D ✓x value ✓ y value (3)

4.6	 area of trapezium AOBC $= \frac{1}{2}(1+2)(1)$ $= 1\frac{1}{2}$ square units area of ΔACD $= \frac{1}{2}(2)(2)$ $= 2$ square units area of quadrilateral OBCD $= 3\frac{1}{2}$ square units $\therefore 2a^2 = \frac{7}{2}$ $a^2 = \frac{7}{4}$ $a = \frac{\sqrt{7}}{2}$	✓ substitution into area of trapezium form ✓ area of trapezium ✓ area of triangle ✓ area of OBCD ✓ equating area OBCD to $2a^2$ (5)



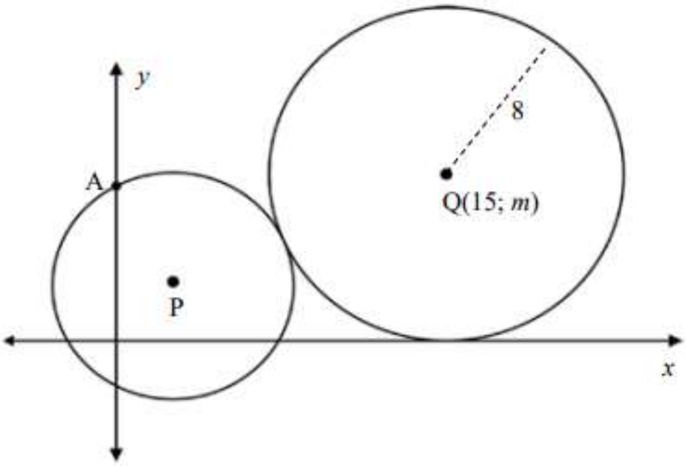
3.1	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5+1)^2 + (13-5)^2}$ $= 10$	✓ use of distance formula/*gebruik afstandformule* ✓ correct subst into form/*korrekte subst in formule* ✓ 10 (3)

3.2	$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{13-5}{5-(-1)}$ $= \frac{8}{6} = \frac{4}{3}$ Answer only: Full marks *slegs antw: volpunte*	✓ correct subst into gradient formula/*korrekte subst in gradiëntformule* ✓ gradient/*gradiënt* (2)

3.3	Equation of line RS/*Vgl van lyn RS*: $m_{RS} = m_{PQ} = \frac{4}{3}$ (= gradients, | | lines/=*gradiënte,* | | *lyne*) $y = mx + c$ $8 = \frac{4}{3}\left(\frac{15}{2}\right) + c$ $c = -2$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $y - 8 = \frac{4}{3}\left(x - \frac{15}{2}\right)$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$	✓ $m_{RS} = \frac{4}{3}$ ✓ subst of S(7,5 ; 8) and m into eq /*subst van S(7,5 ; 8) en m in vgl* ✓ value of c /*waarde van c* or/of st form/*st vorm* ✓ equation/*vgl* (4)
3.4	B is the x-intercept of/*is die x-afsnit van* $y = \frac{4}{3}x - 2$ $0 = \frac{4}{3}x - 2$ $4x - 6 = 0$ $x = \frac{3}{2}$ **OR/*OF*** $4x - 3(0) - 6 = 0$ $4x - 6 = 0$ $x = \frac{3}{2}$	✓ $y = 0$ ✓ $x = \frac{3}{2}$ (2)
3.5	$\tan\ \alpha = \frac{4}{3}$ $\alpha = 53{,}13° = \text{O}\hat{\text{B}}\text{R}$ (vert opp ∠s/*regoorst ∠e*) $\text{O}\hat{\text{R}}\text{B} = 180° - (90° + 53{,}13°)$ (∠s of Δ/*∠e van Δ*) $= 36{,}87°$	✓ $\tan\ \alpha = \frac{4}{3}$ ✓ 53,13° ✓ 36,87° (3)
3.6	$\text{BS} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\frac{15}{2} - \frac{3}{2}\right)^2 + (8 - 0)^2}$ $= 10$ PQ ∥ BS and/*en* PQ = BS PQBS = parallelogram (1 pair opp sides = and ∥/*1 pr tos sye =en* ∥) **OR/*OF*** midpoint of/*midpt van* QS: $\left(\frac{-1+7.5}{2}; \frac{5+8}{2}\right) = \left(\frac{13}{4}; \frac{13}{2}\right)$ midpoint of/*midpt van* PB: $\left(\frac{5+1.5}{2}; \frac{13+0}{2}\right) = \left(\frac{13}{4}; \frac{13}{2}\right)$ PQBS = parallelogram (diags bisect each other/*hoekl halv mekaar)*	✓ correct subst into form/*korrekte subst in formule* ✓ BS = 10 ✓ BS = PQ ✓ reason/*rede* (4) ✓ $\left(\frac{-1+7.5}{2}; \frac{5+8}{2}\right)$ ✓ $\left(\frac{5+1.5}{2}; \frac{13+0}{2}\right)$ ✓ $\left(\frac{13}{4}; \frac{13}{2}\right)$ ✓ reason/*rede* (4)

1.1	$m_{AC} = \frac{-3-1}{5+3} = -\frac{1}{2}$	✓ substitution/*substitusie* ✓ answer/*antwoord*	(2)
1.2	The equation of AC will be/*Die vergelyking AC sal wees*: $y-1=-\frac{1}{2}(x+3)$ $y=-\frac{1}{2}x-\frac{1}{2}$ **OR** $1=-\frac{1}{2}(-3)+c$ $c=-\frac{1}{2}$ $y=-\frac{1}{2}x-\frac{1}{2}$	✓ substitution of gradient/ *substitusie van gradiënt* ✓ substitution of point/ *substitusie van punt* ✓ answer/*antwoord*	(3)
1.3	$\tan\theta=-\frac{1}{2}$ $\theta=153{,}43°$	✓ tan… ✓ gradient/*gradiënt* ✓ answer/*antwoord*	(3)
1.4	$m_{BD}=\frac{4}{x-2}$ $m_{AC}=\frac{-4}{8}=-\frac{1}{2}$ BD ⊥ AC given/*is gegee* i.e./*dit is* $\left(\frac{4}{x-2}\right)\times\left(-\frac{1}{2}\right)=-1$ $\frac{4}{x-2}=2$ $4=2x-4$ $x=4$ D(4 ; 0)	✓ gradient of BD/ *gradiënt van BD* ✓ gradient of AC/ *gradiënt van AC* ✓ product of gradients = –1/ *produk van gradiënte = –1* ✓ value of *x* /*waarde van x* ✓ coordinates of D/ *koördinate van D*	(5)
1.5	$AD=\sqrt{7^2+1^2}=\sqrt{50}$ $AB=\sqrt{(-5)^2+5^2}=\sqrt{50}$ $DC=\sqrt{(-1)^2+3^2}=\sqrt{10}$ $BC=\sqrt{3^2+1^2}=\sqrt{10}$ Two pairs of adjacent sides are equal ∴ ABCD is a kite	✓ any 2 distances/ *enige 2 afstande* ✓ remaining 2/*oorblywende 2* ✓ reason/*rede*	

1.6	$BD=\sqrt{2^2+4^2}=\sqrt{20}=2\sqrt{5}$ M(3 ; –2) $AM=\sqrt{(-6)^2+(3)^2}=\sqrt{45}=3\sqrt{5}$ $\therefore$ Area/Opp. Δ ABD = $\frac{1}{2}$base × height/ $\frac{1}{2}$*basis × hoogte* $=\frac{1}{2}\times(2\sqrt{5})\times(3\sqrt{5})$ = 15 sq. units/*vierkante eenhede*	✓ BD ✓ coordinates of M/ *koördinate van M* ✓ AM ✓ substitution into area formula/*substitusie in oppervlak formule* ✓ answer/*antwoord*	(5)
			[21]

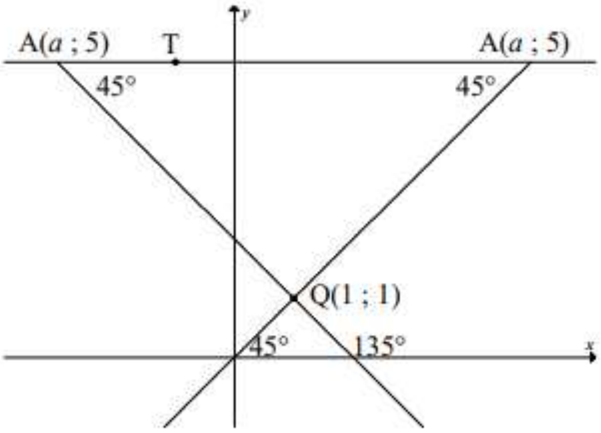


2.1	2.1.1	$(x-15)^2+(y-m)^2=64$	✓answer/*antwoord*	(1)
	2.1.2	(15; 0) lies on circle Q/*lê op sirkel Q* $(15-15)^2+(0-m)^2=64$ $m=8$ **OR/*OF*** $m=y_Q=8$	✓ substitution/ *substitusie* ✓ answer/*antwoord* **OR/*OF*** ✓✓ answer only/ *slegs antwoord*	(2)
	2.1.3	PQ = 8 + 5 = 13 **OR/*OF*** $PQ=\sqrt{(3-8)^2+(3-15)^2}$ = 13 units	✓substitution/ addition/ *substitusie/som van* ✓ answer/*antwoord*	(2)

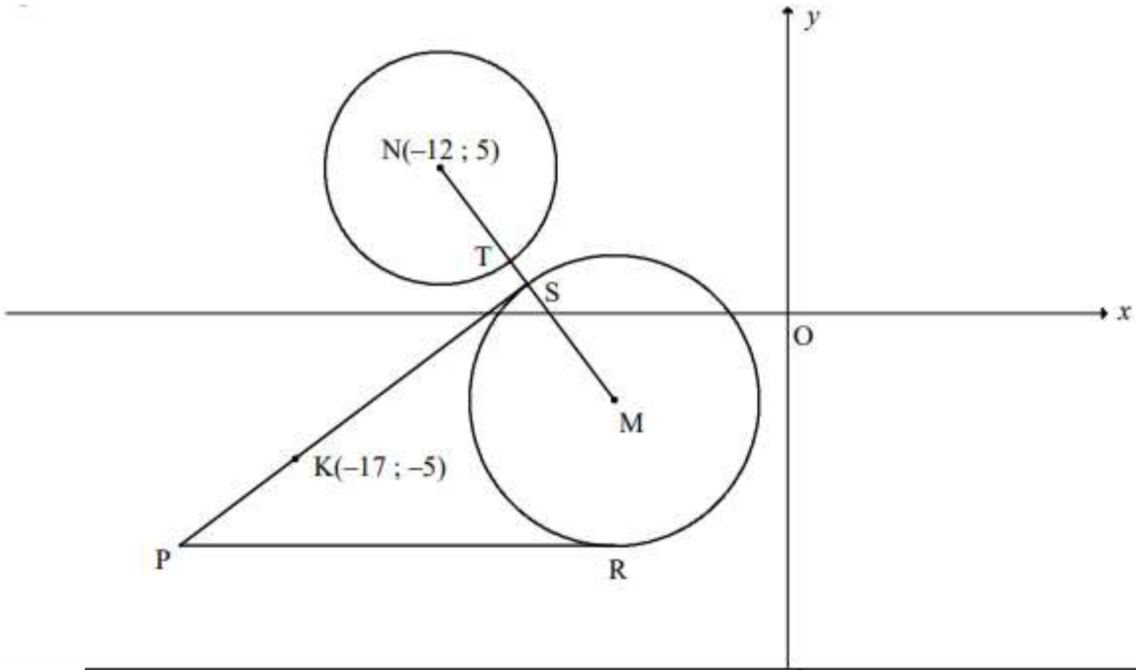
	2.1.3	PQ = 8 + 5 = 13 **OR/*OF*** $PQ=\sqrt{(3-8)^2+(3-15)^2}$ = 13 units	✓substitution/ addition/ *substitusie/som van* ✓ answer/*antwoord*	(2)
	2.1.4	The coordinates of/*Die koördinate van* A(0;y) $(0-3)^2+(y-3)^2=25$ $y=7$ A(0;7) **OR/*OF*** $x_A=0$ $y_A=3+\sqrt{5^2-3^2}=7$ A(0; 7)	✓ substitution/ *substitusie* ✓ *x*-value/*x-waarde* ✓ *y*-value/*y-waarde* **OR/*OF*** ✓ substitution/ *substitusie* ✓ *x*-value/*x-waarde* ✓ *y*-value/*y-waarde*	(3)

	2.1.5	$m_{AP}=\frac{3-7}{3-0}=\frac{-4}{3}$ $m_{rad}\times m_{\tan}=-1$ $m_{\tan}=\frac{3}{4}$ $y=\frac{3}{4}x+7$	✓ m_{AP} ✓ $m_{\tan}$ ✓ product of gradients/*produk van gradiente* ✓ equation/*vergelyking*	(4)
2.2	2.2.1	$(x-3)^2+(y+2)^2=12+9+4$ $(x-3)^2+(y+2)^2=25$ Centre/*middelpunt* is (3; –2) Radius = 5	✓ completing a square/ *voltooiing van vierkant* ✓equation/*vergelyking* ✓ centre/*middelpunt* ✓radius	(4)
	2.2.2	$KT=\sqrt{(12-3)^2+(10+2)^2}$ $=15$	✓substitution/*substitusie* ✓answer/*antwoord*	(2)
	2.2.3	$r_K+r_T=5+10$ $=15=KT$ ∴ the circles intersect at **one** point./*die sirkels sny op* ***een plek***	✓Addition of radii/*som van radiusse* ✓answer/*antwoord*	(2)
				[20]

3.1	K(7 ; 0)	✓ answer (1)
3.2	$1 = \dfrac{x_M + 7}{2}$ *and* $1 = \dfrac{y_M + 3}{2}$ $\therefore$ M(– 5 ; – 1)	✓ x ✓ y (2)
3.3	$m_{PM} = \dfrac{3-1}{7-1}$ $= \dfrac{1}{3}$	✓ substitution ✓answer (2)
3.4	$\tan P\hat{S}K = m_{PM} = \dfrac{1}{3}$ $P\hat{S}K = \tan^{-1}\left(\dfrac{1}{3}\right) = 18{,}43°$ $\therefore\ \theta = 180° - 90° - 18{,}43° = 71{,}57°$	✓ $\tan P\hat{S}K = m_{PM}$ ✓ $P\hat{S}K$ ✓ θ (3)
3.5	$\cos 71{,}57° = \dfrac{PK}{PS} = \dfrac{3}{PS}$ $PS = \dfrac{3}{\cos 71{,}57°}$ $= 9{,}49$ units **OR**	✓ correct ratio ✓ PS as subject ✓ answer (3)
3.6	N(x ; $-2x + 17$) $m_{TN} = m_{PM}$ (TN || PM) $\dfrac{-2x + 17 - 5}{x - (-1)} = \dfrac{1}{3}$ $-6x + 36 = x + 1$ $-7x = -35$ $x = 5$ $\therefore\ y = -2(5) + 17 = 7$ $\therefore$ N(5 ; 7)	✓ N in terms of x ✓ equal gradients ✓ substitution ✓ x-value ✓ y-value (5)

3.7.1	$y = 5$	✓ equation (1)
3.7.2	 gradient of AQ = tan 45° or tan 135° $= 1$ or -1 $m_{AQ} = \frac{5-1}{a-1} = \pm 1$ $\therefore\ a - 1 = 4$ or -4 $\therefore\ \ a = 5$ or -3	✓ $m_{AQ} = 1$ or ✓ $m_{AQ} = -1$ ✓ substitution into gradient formula ✓ x-value ✓ y-value (5) **[22]**

QUESTION 6

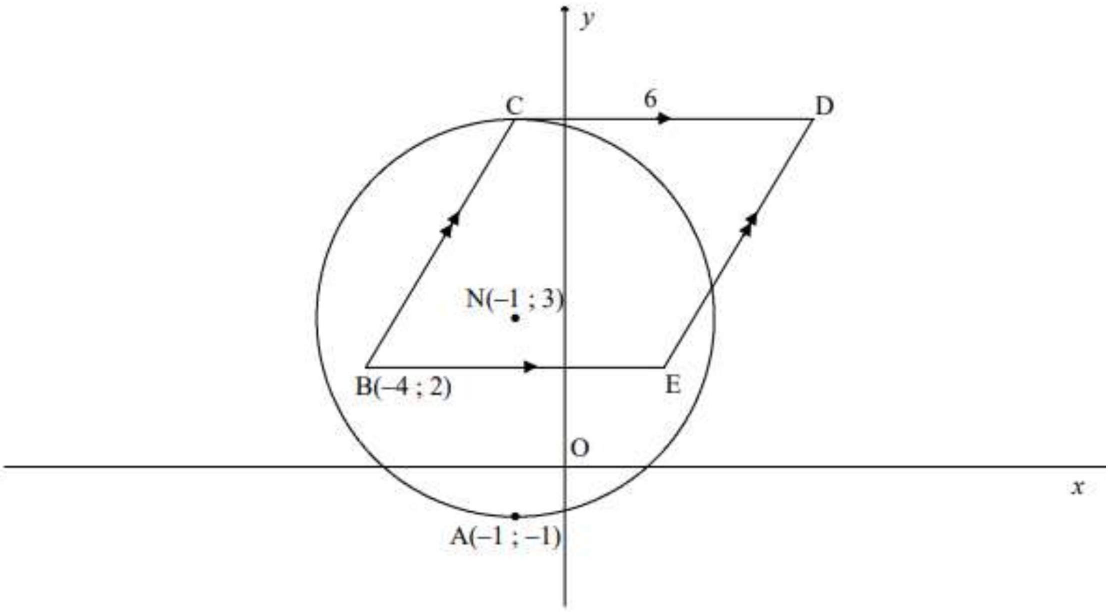


6.1	$M(-6;-3)$	✓ –6 ✓ –3 (2)
6.2.1	$^2+y^2+24x-10y+153=0$ $x+12)^2+(y-5)^2=-153+144+25$ $(x+12)^2+(y-5)^2=16$ $r^2=16$ $r=4$ units	✓ $r^2=-153+144+25$ ✓ length of radius (2)
6.2.2	$M=\sqrt{(-12-(-6))^2+(5-(-3))^2}$ NM = 10 units SM = 5 units $\therefore$ TS = 10 – 5 – 4 = 1 unit	✓ substitution into distance formula ✓ NM = 10 units ✓ SM = 5 units ✓ answer (4)
6.2.3	–6 ; –8) = –8	✓ $y_R=-8$ ✓ answer (2)

6.3.2	$m_{NM} = \frac{5-(-3)}{-12-(-6)}$ $m_{NM} = -\frac{4}{3}$ $m_{tangent} = \frac{3}{4}$ $-5 = \frac{3}{4}(-17)+c$ **OR/*OF*** $y - y_1 = \frac{3}{4}(x - x_1)$ $c = \frac{31}{4}$ $y-(-5) = \frac{3}{4}(x-(-17))$ $y = \frac{3}{4}x + \frac{31}{4}$ $y = \frac{3}{4}x + \frac{31}{4}$	✓ substitution ✓ $m_{NM} = -\frac{4}{3}$ ✓ $m_{tangent} = \frac{3}{4}$ ✓ substitution of m and N ✓ equation (5)
6.4.	$-8 = \frac{3}{4}x + \frac{31}{4}$ $-32 = 3x + 31$ $3x = -63$ $x = -21$ P(–21 ; –8) R(–6 ; –8) PR = PS = 15 units [tangents from same point] MS = MR = 5 units Perimeter PSMR = 15 + 15 + 5 + 5 = 40 units	✓ $-8 = \frac{3}{4}x + \frac{31}{4}$ ✓ $x = -21$ ✓ PR = PS = 15 units ✓ MS = MR = 5 units ✓ answer (5)

6.4.2	$\dfrac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ $\dfrac{\frac{1}{2}NS.SP}{\frac{1}{2}SP.MS+\frac{1}{2}MR.PR}$ $=\dfrac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$ $=\dfrac{1}{2}$ **OR** $\Delta NPS \equiv \Delta SPM \equiv \Delta MPR$ $\dfrac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ $=\dfrac{1}{2}$	✓ substitution ✓ answer (2) ✓ congruent ✓ answer (2)
		[22]

Question 7

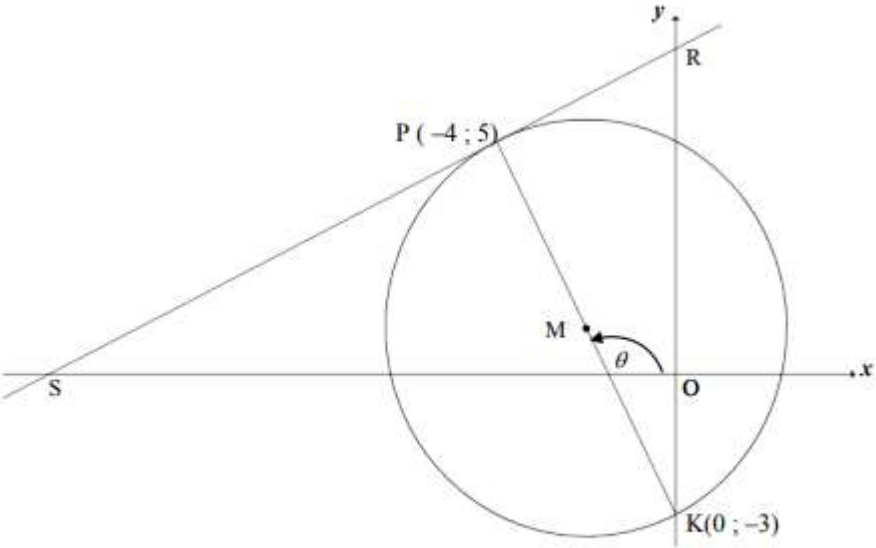


7.1	Radius = 4 units/*eenhede*	✓ answer (1)
7.2.1	$CD \perp CN$ $\therefore C(-1\,;\,7)$	✓x value ✓ y value (2)
7.2.2	D = 6 units D(5 ; 7)	✓x value ✓ y value (2)

7.2.3	$\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ **OR/*OF*** $\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}[\text{Area of } \parallel^{m}]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

7.3.1	M(3 ; −1) [reflection of N(−1 ; 3) about the line $y = x$] $\therefore \text{MN} = \sqrt{(3-(-1))^2 + (-1-3)^2}$ $\text{MN} = \sqrt{32} = 4\sqrt{2} = 5{,}66$ units	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
7.3.2	M(3 ; −1) $m_{\text{MN}} = \frac{3-(-1)}{-1-3} = -1$ MN: $-1 = -(3) + c$ or $y - 3 = -1(x+1)$ $c = 2$; $y - 3 = -x - 1$ $\therefore y = -x + 2$; $y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint (1 ; 1)	✓ equation of MN ✓ equating AF & MN ✓x value ✓ y value (4)

Question 8



8.1.	$m_{PK} = \frac{5-(-3)}{-4-0}$ $= -2$ PK $\perp$ SR [radius $\perp$ tangent/*raaklyn*] $\therefore\ m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	✓ substitution P & K into gradient formula ✓ gradient of PK ✓ PK $\perp$ SR **OR** r $\perp$ tangent ✓ answer (4)

8.1.2	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c$ **OR/*OF*** $(y-5) = \frac{1}{2}(x-(-4))$ $c = 7$ $(y-5) = \frac{1}{2}x + 2$ $y = \frac{1}{2}x + 7$ $y = \frac{1}{2}x + 7$	✓ substitution of *m* and P ✓ equation (2)

8.1.	$M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)$ $\therefore M(-2;1)$ $r^2=(x_2-x_1)^2+(y_2-y_1)^2$ $r^2=(-2+4)^2+(1-5)^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20\ or\ (\sqrt{20})^2$ **OR/*OF*** $M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)\ \therefore M(-2;1)$ $(x+2)^2+(y-1)^2=r^2$ $(-4+2)^2+(5-1)^2=r^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20\ or\ (\sqrt{20})^2$ **OR/*OF*** $M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)\ \therefore M(-2;1)$ $PK=\sqrt{(-4-0)^2+(5-(-3))^2}=\sqrt{80}$ $r=\frac{\sqrt{80}}{2}=\sqrt{20}$ $\therefore (x+2)^2+(y-1)^2=20\ or\ (\sqrt{20})^2$	✓ x value of M ✓ y value of M ✓ $r^2=20$ ✓ equation (4) ✓✓ M $(-2;1)$ $r^2=20$ ✓ equation (4) ✓✓ M $(-2;1)$ $r^2=20$ ✓ equation (4)

8.1.	$\tan\theta=m_{PK}=-2$ $\therefore \theta=180°-63,43°$ $=116,57°$ $P\hat{K}R=116,57°-90°$ [ext $\angle$of ΔMOK] $=26,57°$	✓ $\tan\theta=-2$ ✓ size of θ ✓ answer (3)

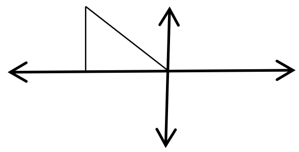
8.1.	RS ∥ tangent at K(0 ; −3) $\therefore m_{PS} = m_{tang} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$ **OR/*OF*** $m_{PK} = \frac{1-5}{-2+4} = -2$ $m_{PK} \times m_{tan\,g} = -1$ [radius ⊥ tangent/*raaklyn*] $\therefore m_{tan\,g} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$	✓ gradient ✓ equation (2) ✓ gradient ✓ equation (2)
8.2	$t \in (-3\ ;\ 7)$ **OR/*OF*** $-3 < t < 7$	✓ −3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3) ✓ −3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3)
8.3	RS: $y = \frac{1}{2}x + 7$ ∴ S(−14 ; 0) $SP = \sqrt{(-14-(-4))^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125}$ Area ΔSMK = $\frac{1}{2}$. MK . SP $= \frac{1}{2}(\sqrt{20})(\sqrt{125})$ = 25 square units	✓ coordinates of S ✓ length of SP ✓ correct base & height into Area rule ✓ correct substitution ✓ answer (5)

TRIG RATIO, IDENTITIES, REDUCTIONS, AND GENERAL SOLUTIONS

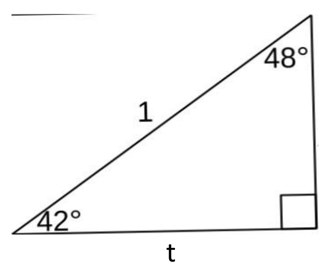
QUESTION 5 A

No.	Solution	Marks
5.1.1	$tan\beta = \frac{\sqrt{5-p^2}}{p}$	(4)
5.1.2	$\cos 2\beta$ $= 2\cos^2\beta - 1$ $= \mathbf{2\left(-\frac{p}{\sqrt{5}}\right)^2 - 1}$ $= \frac{2p^{\ 2}}{5} - 1$	(3)

5.2.1	$17\cos\beta + 15 = 0$ $\text{Cos}\ \beta = \frac{-15}{17}$ $x^2 + y^2 = r^2$ $m^2 = 17^2 - 15^2$ $= 64$ $m = -8$	(3)
5.2.2	$\sin\beta + \tan\beta = \frac{\mathbf{-8}}{\mathbf{17}} + \frac{-8}{-15}$ $= \frac{16}{255}$	(3)
5.2.3	$\text{Cos}\ 2\beta = 2\cos^2\beta - 1$ $= 2(\frac{-15}{17})^2 - 1$ $= \frac{161}{289}$	(3)

		
5.3.1	$\tan \alpha$ $= \quad \frac{8}{15}$	(3)
5.3.2	$\sin(90° + \alpha\ )$ $= \cos \alpha$ $= \quad \frac{15}{17}$	(2)

5.3.3	$\cos 2\alpha$ $= 1 - 2\sin^2 x$ $= 1 - 2\ (\frac{8}{17})^2$ $\frac{161}{289}$	(3)

5.4.1	$\tan\theta = p$	(1)
5.4.2	$\tan 120^0 = \frac{\sqrt{3}}{p}$ $\tan(180^0 - 60^0) = \frac{\sqrt{3}}{p}$ $-\tan 60^0 = \frac{\sqrt{3}}{p}$ $-\sqrt{3} = \frac{\sqrt{3}}{p}$ $P = -1$	(3)
5.5.1	$\cos 42^0$ $= \sin 48^0$ $= t$  	(2)
5.5.2	$\cos 2(42^0)$ $= 2\cos^2 42^0 - 1$ $=- 2t^2 - 1$ OR $\cos 2(42^0)$ $= 1 - 2\sin^2 42^0$ $= 1 - 2\left(\sqrt{1 - t^2}\right)^2$ $= 2t^2 - 1$	

5.5.2	OR COS 2 (42^0) $= \cos^2 42^0 — \sin^2 42^0$ $= t^2 - (\sqrt{1-t^2})^2$ $= t^2 - (1 - t^2)$ $= 2t^2 - 1$	(3)
5.5.3	Cos $(42^0 + 30^0)$ $= \cos 42^0 . \cos 30^0 — \sin 42^0 . \sin 30^0$ $= t . \frac{\sqrt{3}}{2} - (\sqrt{1-t^2}) . \frac{1}{2}$ $= \frac{t\sqrt{3}}{2} - \frac{(\sqrt{1-t^2})}{2}$ $= \frac{\sqrt{3}t - \sqrt{1-t^2}}{2}$	(5)
5.6.1	Cos 214^0 $= \cos (180^0 + 34^0)$ $= -\cos 34^0$ $= -p$	**(2)**

5.6.2	Cos 68^0 $= \cos 2(34^0)$ $=2\cos^2 34^0 - 1$ $=2p^2 - 1$	(2)

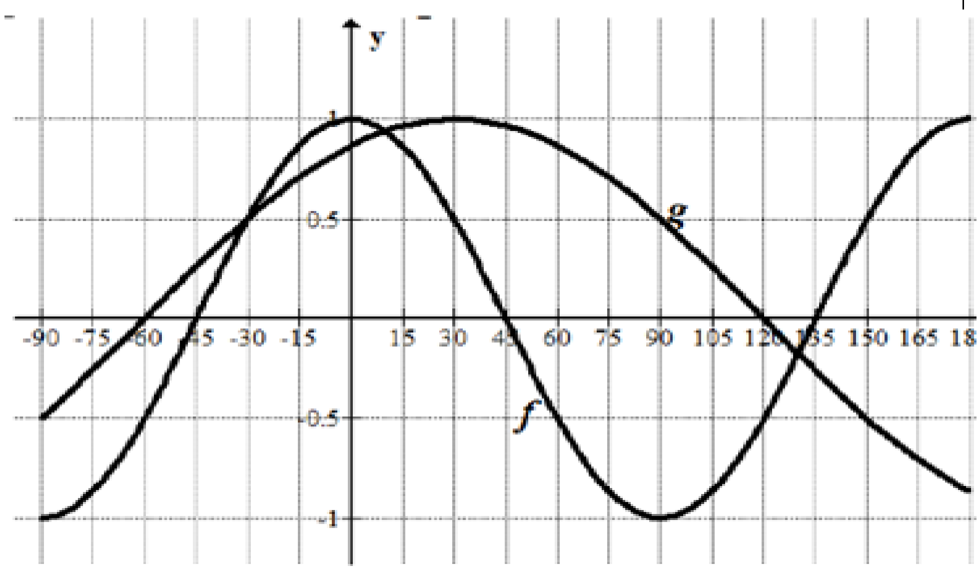
TRIGONOMETRIC GENERAL SOLUTIONS

5.10.1	$\cos 2x = 1 - 3\cos x$ $2\cos 2x - 1 + 3\cos x - 1 = 0$ $2\cos 2x + 3\cos x - 2 = 0$ $(2\cos x - 1)(\cos x + 2) = 0$ $\cos x = \frac{1}{2}$ or $\cos x = -2$ $x_{ref} = 60°$ or No solution <u>First quadrant:</u> $x = 60° + 360°k,\ k \in \mathbb{Z}$ <u>Fourth quadrant:</u> $x = 300° + 360°k,\ k \in \mathbb{Z}$	(6)
5.10.2	$\frac{\tan x - 1}{2} = 3$ $\tan x - 1 = -6$ $\tan x = -5$ $x\,ref = 78{,}69°$ <u>Second quadrant:</u> $x = 101{,}31° + 180°k,\ k \in \mathbb{Z}$ <u>Fourth quadrant:</u>	(5)

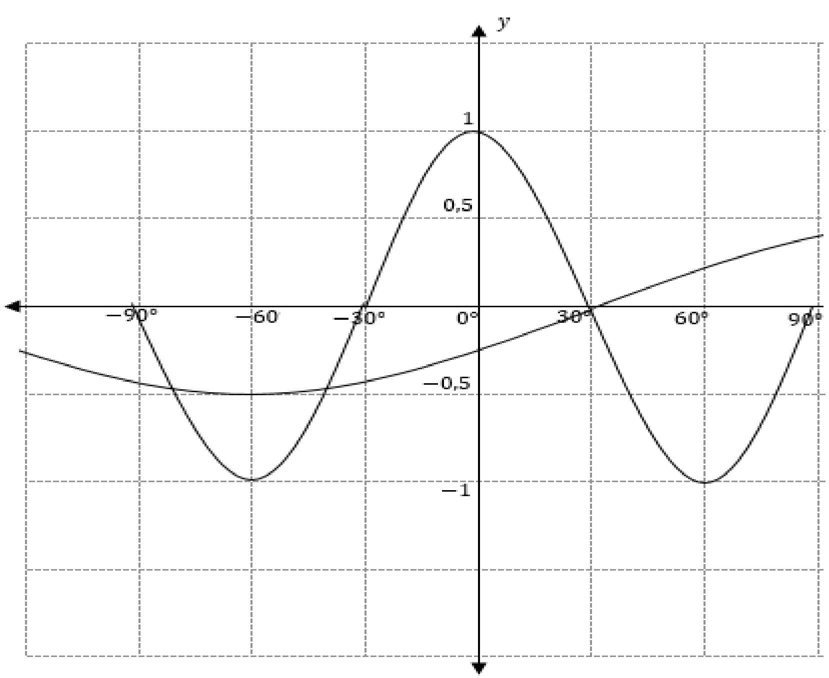
	$x = 281{,}31° + 180°k\ ,\ k \in \mathbb{Z}$		
5.10.3	$\sin x + 2\cos^2 x = 1$ $\sin x + 2(1 - \sin^2 x) - 1 = 0$ $-2\sin^2 x + \sin x + 2 - 1 = 0$ $2\sin^2 x - \sin x - 1 = 0$ $(2\sin x + 1)(\sin x - 1) = 0$ $\sin x = -\frac{1}{2}$ or $\sin x = 1$ $x_{ref} = 30°$ <u>Third quadrant:</u> $x = 210° + 360°k\ ,\ k \in \mathbb{Z}$ <u>Fourth quadrant:</u> $x = 330° + 360°k\ ,\ k \in \mathbb{Z}$	$\sin x = 1$ $x_{ref} = 90°$ $x = 90° + 360°k\ ,\ k \in \mathbb{Z}$	(7)
5.10.4	$6\cos x - 5 = \frac{4}{\cos x}$; $\cos x \neq 0$ $6\cos^2 x - 5\cos x - 4 = 0$ $(2\cos x + 1)(3\cos x - 4) = 0$ $\cos x = -\frac{1}{2}$ or $\cos x = \frac{4}{3}$ $x_{ref} = 60°$ or No solution <u>Second quadrant:</u> $x = 120° + 360°k\ ,\ k \in \mathbb{Z}$ <u>Third quadrant:</u> $x = 240° + 360°k\ ,\ k \in \mathbb{Z}$		(6)
5.10.5	$\sin 2x = \cos(x - 30°)$ $\sin 2x = \sin[90° - (x - 30°)]$ $\sin 2x = \sin(120° - x)$ $2x = 120° - x + 360°k\ ,\ k \in \mathbb{Z}$ $3x = 120° + 360°k\ ,\ k \in \mathbb{Z}$ $x = 40° + 120°k\ ,\ k \in \mathbb{Z}$ OR		(6)

	$2x = [180 - (120° - x)] + 360°\,k\,,\ k\ \in \mathbb{Z}$ $x = 60° + 360°\,k\,,\ k\ \in \mathbb{Z}$	

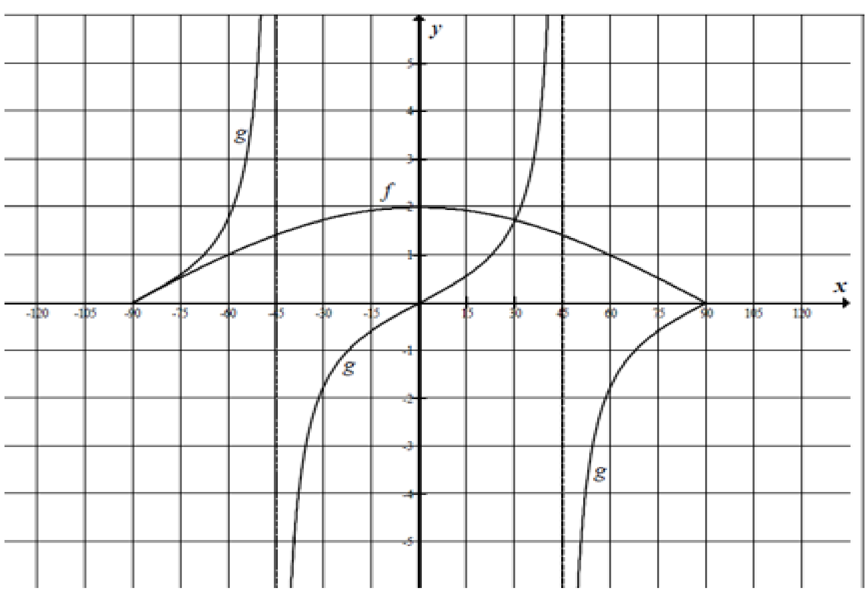
TRIGONOMETRIC FUNCTIONS

<table>
<tr><td colspan="3">ACTIVITY 6A</td></tr>
<tr><td>1.1.</td><td>


</td><td>(6)</td></tr>
<tr><td>1.2.</td><td>Period = 240°</td><td>(1)</td></tr>
<tr><td>1.3.</td><td>$h(x) = \text{Cos}\,(2x - 90°) - 1$
$h(x) = \sin 2x - 1$</td><td>(2)</td></tr>
<tr><td>1.4.</td><td>$\text{Sin}\,(x + 60°) = \sin\,(90° - 2x)$
$x + 60° = 90° - 2x + 360°k\,,\ k\ \in \mathbb{Z}$
$3x = 30° + 360°k\,,\ k\ \in \mathbb{Z}$
$x = 10° + 120°k\,,\ k\ \in \mathbb{Z}$
Or
$x + 60° = 180 - (90° - 2x) + 360°k\,,\ k\ \in \mathbb{Z}$
$x = -30° + 360°k\,,\ k\ \in \mathbb{Z}$</td><td>(5)</td></tr>
</table>

	$\therefore x \in \{-30°; 10°; 130°\}$	
ACTIVITY 6B		
1.1.	$Maximum\ value = 4$	(1)
1.2.	$Range{:}\ -3 \leq y \leq 5\ or\ y \in [-3;\ 5]$	(2)
1.3.	$h(x) = -g(x + 60°)$ $= -[-4\cos(x + 30° + 60°)]$ $= -[-4\cos(90° + x)]$ $= -[4 sinx]$ $= -4 sinx$	(3)
ACTIVITY 6C		
1.1.	$Period\ = 120°$	
1.2.		
1.3.1.	⍰ ∈ (30°; 90°)	(2)
1.3.2.	⍰ ∈ (−90°; −30°)	(2)
1.4.	Range: $-4 \leq$ ⍰ $\leq \frac{1}{2}$	(2)
2.1.	$f(x) = g(x)$	(7)

	$1 + \sin x = 1 - 2\sin2x$ $2\sin2x + \sin x = 0$ $\sin x\ (2\sin x + 1) = 0$ $\sin x = 0$ or $\sin x = -\frac{1}{2}$ $x = 0°$ or $x = -30° + 360°k,\ k \in \mathbb{Z}$ $x = 180°k$ or $x = 210° + 360°k,\ k \in \mathbb{Z}$ $x \in \{180°; 210°; 330°; 360°\}$	
2.2.	$180° \leq x \leq 210°$ Or $330° \leq x \leq 360°$	(3)
ACTIVITY 6D		
1.1.	$period = 720°$	(1)
1.2.	$p = \frac{1}{2}\ and\ q = 30°$	(2)
1.3.	$-360° \leq x \leq -210°$	(3)
2.1.	$Range: -1 \leq y \leq 1\ or\ y \in [-1; 1]$	(1)
2.2.	$h(x) = 2\sin\left(\frac{1}{2}x\right)$ $\therefore Period = 720°$	(1)
2.3.		
2.3.1.	$x = 45°\ or\ x = 225°$	(2)
2.3.2	$-90° < x < -60°\ or\ 90° < x < 120°$	(2)
ACTIVITY 6E		
1.1.	$Period = 720°$	(1)
1.2.	$Range: -2 \leq y \leq 2\ or\ y \in [-2; 2]$	(2)
1.3		

1.3.1	$x - intercepts\ of\ g\ at - 90° + 60° = -30°$ and $90° + 60° = 150°$ $\therefore x \in (-30°; 150°)$ **OR** $x - intercepts\ of\ g\ at - 90° + 60° = -30°$ and $90° + 60° = 150°$ $-30° < x < 150°$	(3)
1.3.2.	$x \in$ [-180°;-120°) $\cup (-30°; 60°) \cup (150°; 180°]$ **OR** $180° \leq x < -120°\ or\ -30° < x < 60°\ or\ 150° \leq 180°$	(4)
1.4.	$f(-120°) - g(-120°)$ $= -3sin\left(-\frac{120°}{2}\right) - 2\cos(-120° - 60°)$ $= \frac{4 + 3\sqrt{3}}{2}\ or\ 4{,}60\ (4{,}5980 \ldots)$	(3)
ACTIVITY 6F		
1.1.		(8)
1.2.	$0° < \square < 45°$ or $-90° < \square < -45°$	(2)
1.3.	$Period = 2(360°) = 720°$	(2)

1.4.	□ = −45° + 25° = −20° and □ = 45° + 25° = 70°	(2)
2		
2.1.	□ = 135° □ = −45°	(2)
2.2.	□ is shifted by 45° to the left.	(2)
2.3.	g(□) = 3 sin 2□	(2)
ACTIVITY 6G		
1.1.	$f(x) = a \tan x$ $2 = a \tan 225°$ $a = 2$ $g(x) = b \cos x$ $-4 = b \cos 180°$ $-4 = -b$ $b = 4$	(4)
1.2.	$Minimum\ value = -2$	(2)
1.3.	$Period = \frac{180°}{\frac{1}{2}}$ $= 360°$	(2)
1.4.	$2 \tan x = 4 \cos x$ $\frac{\sin x}{\cos x} = 2 \cos x$ $\sin x = 2\cos 2x$ $\sin x = 2(1 - \sin^2 x)$ $2\sin^2 x + \sin x - 2 = 0$ $\sin x = \frac{-1 \pm \sqrt{17}}{4}$ $x = 51{,}33°$ or $x = 128{,}67°$ $x_Q = 180° - x_P$	(4)
ACTIVITY 6H		

6.1.(a)	A (120°; 0)	(1)
6.1.(b)	C (-150°; -1)	(2)
6.2.(a).	$x \in (-90°; 30°)\ OR\ -90° < x < 30°$	(2)
6.2.(b).	$x \in (-160°; 20°)\ OR\ -160° < x < 20°$	(2)
6.3.	$y = 2^{2\sin x + 3}$ $Range\ of\ y = 2\sin x : y \in [-2; 2]\ or\ -2 \leq y \leq 2$ $Range\ of\ y = 2\sin x + 3 : y \in [1; 5] or\ 1 \leq y \leq 5$ $Range\ of\ y = 2^{2\sin x + 3} : y \in [2; 32]\ or\ 2 \leq y \leq 32$	(5)

TRIGONOMETRY 2D AND 3D SOLUTIONS LEVEL 3 & 4

QUESTION 7A		
7.1	In ΔHLB $\tan 40° = \dfrac{3}{LB}$ $\therefore LB = 3{,}58\ m$	
7.2	In ΔABL $AB^2 = AL^2 + LB^2 - 2(AL)(LB)\cos \hat{L}$ $AB^2 = (5{,}2)^2 + (3{,}58)^2 - 2(5{,}2)(3{,}58)\cos 113°$ $AB = 7{,}38\ m$	
7.3	In ΔABL $Area_{\Delta ABL} = \dfrac{1}{2} AL \,.\, LB \sin \hat{L}$ $Area_{\Delta ABL} = \dfrac{1}{2}(5{,}2)(3{,}58)\sin 113°$ $Area_{\Delta ABL} = 8{,}57\ m^2$	
QUESTION 7B		
7.1	$\sin 2x = \dfrac{AB}{r}$ $AB = r \sin 2x$	
7.2	$A\hat{K}C = 90° + x$	

7.3	In ΔABC $BC = r\cos 2x$ $AB = r\sin 2x$ In ΔACK $\dfrac{\sin(90° + x)}{r} = \dfrac{\sin x}{AK}$ $AK = \dfrac{r\sin x}{\cos x}$ $\dfrac{\left(\frac{r\sin x}{\cos x}\right)}{r\sin 2x} = \dfrac{2}{3}$ $\dfrac{r\sin x}{\cos x} \times \dfrac{1}{r2\sin x\cos x} = \dfrac{2}{3}$ $\dfrac{1}{2\cos^2 x} = \dfrac{2}{3}$ $\cos^2 x = \dfrac{3}{4}$ $\cos x = \sqrt{\dfrac{3}{4}}$ $x = 30°$	

QUESTION 7C

	In ΔABC $\frac{\sin[180° - (\alpha + \beta)]}{b} = \frac{\sin\alpha}{BC}$ $\frac{\sin(\alpha + \beta)}{b} = \frac{\sin\alpha}{BC}$ $BC = \frac{b\sin\alpha}{\sin(\alpha + \beta)}$ $DF = BC$ In ΔDEF $\cos\theta = \frac{DF}{DE}$ $DE = \frac{DF}{\cos\theta}$ $DE = \frac{b\sin\alpha}{\sin(\alpha + \beta)} \div \cos\theta$ $DE = \frac{b\sin\alpha}{\sin(\alpha + \beta)\cos\theta}$	
7.2	$DE = \frac{(2\,000)\sin 43°}{\sin(43° + 36°)\cos 27°}$ $DE = 1\,559{,}50\ m$	
QUESTION 7D		

7.1	In ΔKML $\frac{KL}{\sin 70°} = \frac{x}{\sin 42°}$ $KL = \frac{x \sin 70°}{\sin 42°}$ In ΔPKL $\frac{KP}{\sin 35°} = \frac{KL}{\sin 110°}$ $KP = KL \times \frac{\sin 35°}{\sin 110°}$ $KP = \frac{x \sin 70°}{\sin 42°} \times \frac{\sin 35°}{\sin(180° - 70°)}$ $KP = \frac{x \sin 70°}{\sin 42°} \times \frac{\sin 35°}{\sin 70°}$ $KP = \frac{x \sin 35°}{\sin 42°}$	
7.2	$Area\ \Delta KPL = \frac{1}{2} KP.KL.\sin \hat{P}$ $= \frac{1}{2}\left(\frac{70 sin35}{sin42}\right)\left(\frac{70 sin35}{sin42}\right) sin110$ $= 1691.7\ m^2$	
QUESTION 7E		
7.1	In ΔCBD $\frac{CB}{\sin 2x} = \frac{k}{\sin(90° - x)}$ $\frac{CB}{\sin 2x} = \frac{k}{\cos x}$ $CB = \frac{k \cdot 2 \sin x \cos x}{\cos x}$	

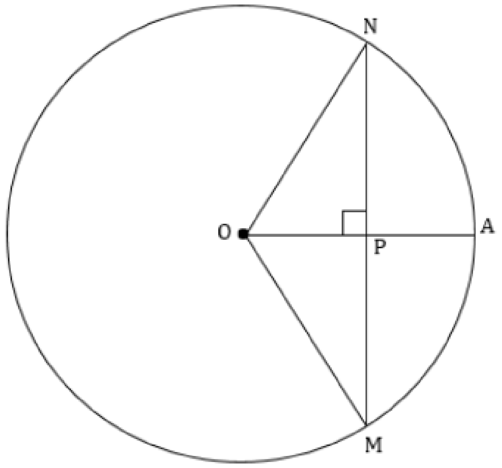
	$CB = 2k \sin x$	
7.2	In ΔBCH In ΔBCH $\cos x = \dfrac{2k \sin x}{HC}$ $HC = \dfrac{2k \sin x}{\cos x}$ $HC = 2k \tan x$	
7.3	$\cos \theta = \dfrac{(80 \tan 23°)^2 + (3{,}18)^2 - (40)^2}{2(80 \tan 23°)(31{,}8)}$ $\theta = 74{,}5°$	
QUESTION 7F		
7.1	In ΔQRT $\dfrac{TR}{\sin 60°} = \dfrac{k}{\sin[180° - (60° + \theta)]}$ $\dfrac{TR}{\sin 60°} = \dfrac{k}{\sin(60° + \theta)}$ $TR = \dfrac{k \sin 60°}{\sin(60° + \theta)}$	

7.2	ΔRST $\sin 60° = \dfrac{RS}{TR}$ $RS = TR \sin 60°$ $RS = \dfrac{k \sin 60°}{\sin(60° + \theta)} \times \sin 60°$ $RS = \dfrac{k\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{\sin 60° \cos\theta + \cos 60° \sin\theta}$ $RS = \dfrac{\left(\frac{3}{4}\right)k}{\left(\frac{\sqrt{3}}{2}\right)\cos\theta + \left(\frac{1}{2}\right)\sin\theta}$ $RS = \dfrac{\left(\frac{3}{4}\right)k}{\frac{1}{2}(\sqrt{3}\cos\theta + \sin\theta)}$ $RS = \dfrac{3k}{2(\sqrt{3}\cos\theta + \sin\theta)}$	

GRADE 12

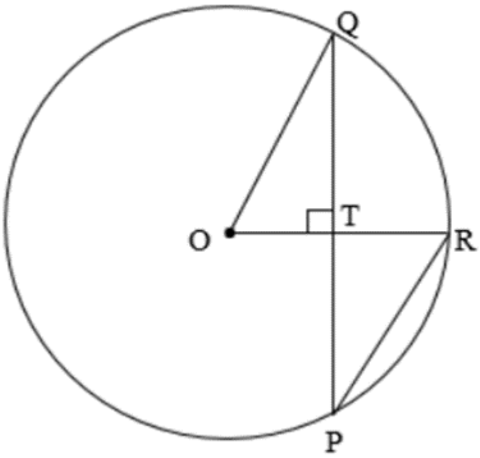
LEVEL 1 AND LEVEL 2 SOLUTIONS

QUESTION 1



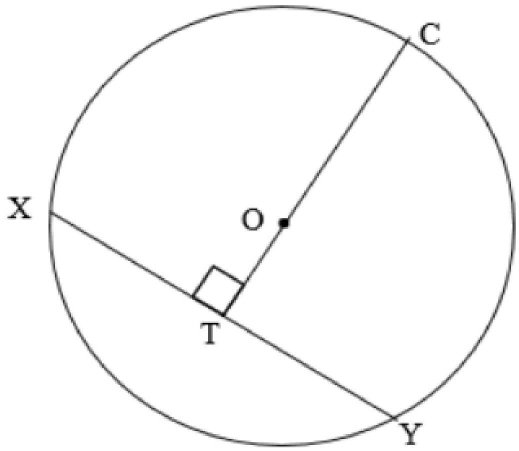
STATEMENT	REASON
$MP = 24$ units	Line from centre $\perp$ chord
In ΔOPM	Right-angled triangle
$OM^2 = MP^2 + OP^2$	Pythagoras theorem
$OM^2 = 24^2 + 7^2$	
$OM = 25$ units	Equal radii
$OA = OM$	
$OP + PA = OA$	
$7 + PA = 25$	
$PA = 18$ units	

QUESTION 2



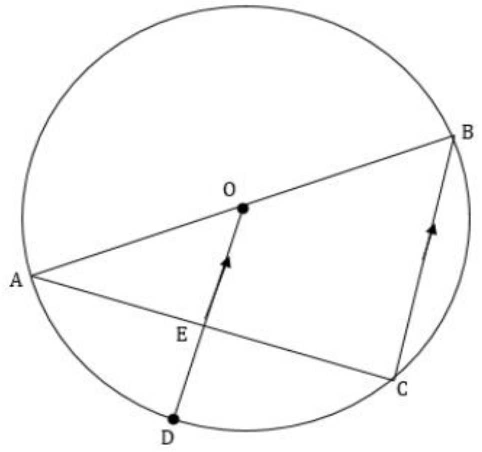
STATEMENT	REASON
2.1 $PT + TQ = PQ$ $PQ = 24$ units	 Line from centre $\perp$ chord
2.2 In ΔOTQ $OQ^2 = QT^2 + OT^2$ $13^2 = 12^2 + OT^2$ $OT = 5$ units $TR = 8$ units In ΔPTR $PR^2 = TR^2 + TP^2$ $PR^2 = 8^2 + 12^2$ $PR = 4\sqrt{13}$ units	 Right-angled triangle Pythagoras theorem Right-angled triangle Pythagoras theorem

QUESTION 3



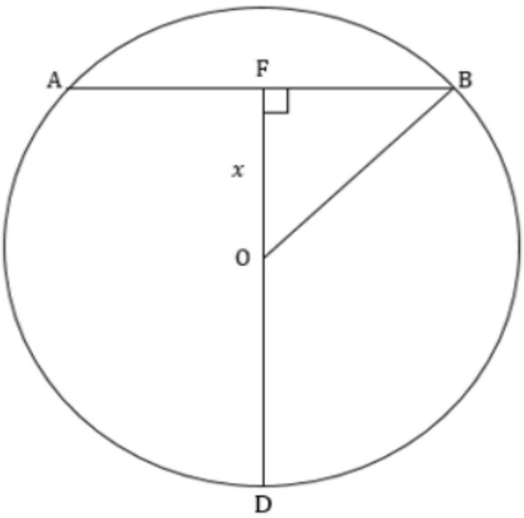
STATEMENT	REASON
In ΔOTY	Right-angled triangle
$TY = \frac{3}{4}r$	Line from centre $\perp$ chord
$OY^2 = TY^2 + OT^2$	Pythagoras theorem
$r^2 = \left(\frac{3}{4}r\right)^2 + OT^2$	
$r = \frac{\sqrt{7}}{4}r$	Given
$CT = OT + OC$	
$CT = \frac{\sqrt{7}}{4}r + r$	
$CT = r\left(\frac{\sqrt{7}}{4} + 1\right)$	
$CT = r\left(\frac{\sqrt{7}}{4} + \frac{4}{4}\right)$	
$\therefore CT = \frac{4+\sqrt{7}}{4}r$	

QUESTION 4



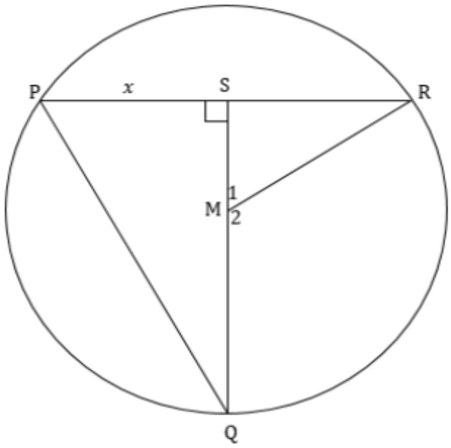
STATEMENT	REASON
$AE = 8$ cm	Line from centre $\perp$ chord
In ΔAOE	Right-angled triangle
$AO^2 = AE^2 + OE^2$	Pythagoras theorem
$10^2 = 8^2 + OE^2$	
$OE = 6$ cm	
$OD = AO$	Radii
$ED + OE = OD$	
$ED + 6 = 10$	
$ED = 4$ cm	

QUESTION 5



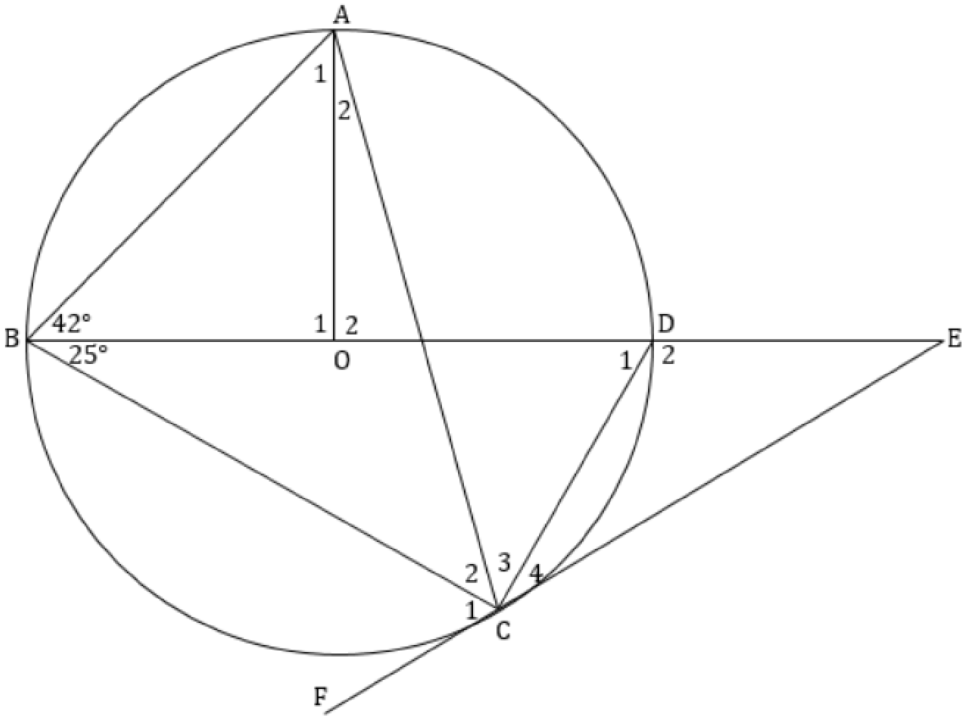
STATEMENT	REASON
$FB = 4$ cm	Line from centre $\perp$ chord
$OD = 8 - x$	
$OB = OD$	Radii
In ΔOBF	Right-angled triangle
$OB^2 = FD^2 + FB^2$	Pythagoras theorem
$(8 - x)^2 = x^2 + FB^2$	
$FB^2 = 64 - 16x + x^2 - x^2$	
$FB^2 = 64 - 16x$	
$4^2 = 64 - 16x$	
$x = 3$ cm	
$OD = 8 - 3$	

QUESTION 6



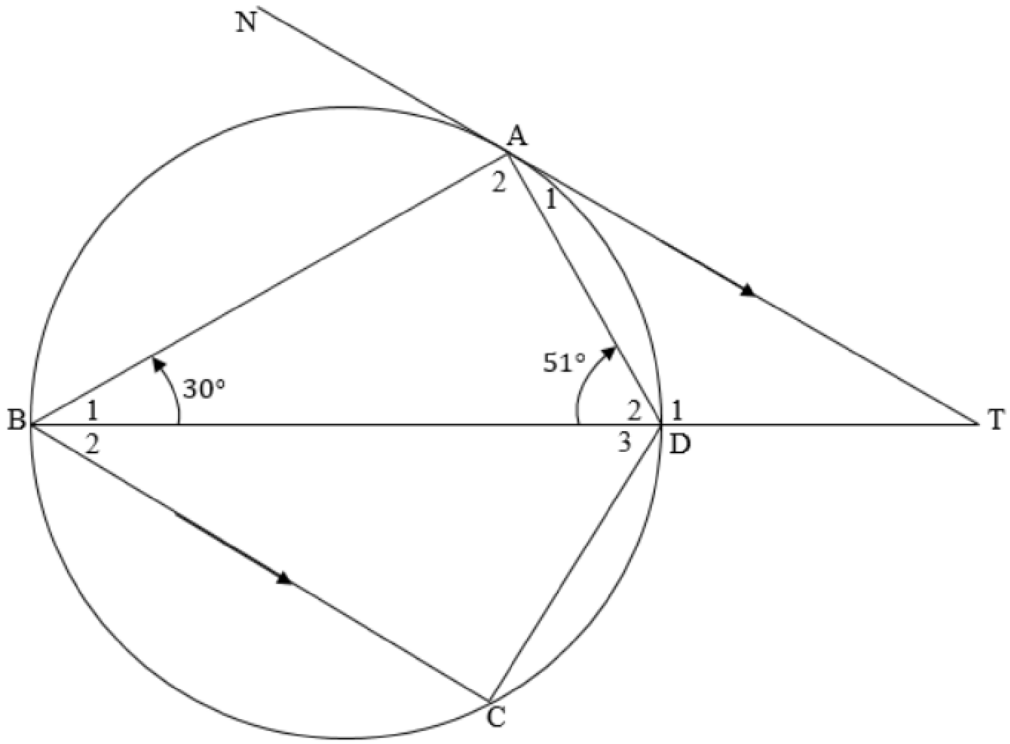
STATEMENT	REASON
6.1 $SR = x$ In ΔOBF $MR^2 = MS^2 + SR^2$ $r^2 = MS^2 + x^2$ $MS^2 = r^2 - x^2$ $MS = \sqrt{r^2 - x^2}$ $QS = r + \sqrt{r^2 - x^2}$	 Line from centre $\perp$ chord Right-angled triangle Pythagoras theorem
6.2 $1 = \sqrt{r^2 - (\sqrt{12})^2}$ $1 = r^2 - 12$ $r = \sqrt{13}$ units	
6.3 In ΔPSQ $\tan \hat{P} = \frac{QS}{PS}$ $\tan \hat{P} = \frac{1 + \sqrt{13}}{\sqrt{12}}$ $\hat{P} = \tan^{-1}\left(\frac{1 + \sqrt{13}}{\sqrt{12}}\right)$ $\hat{P} = 53{,}05°$	 Right-angled triangle Trigonometric ratios

QUESTION 7



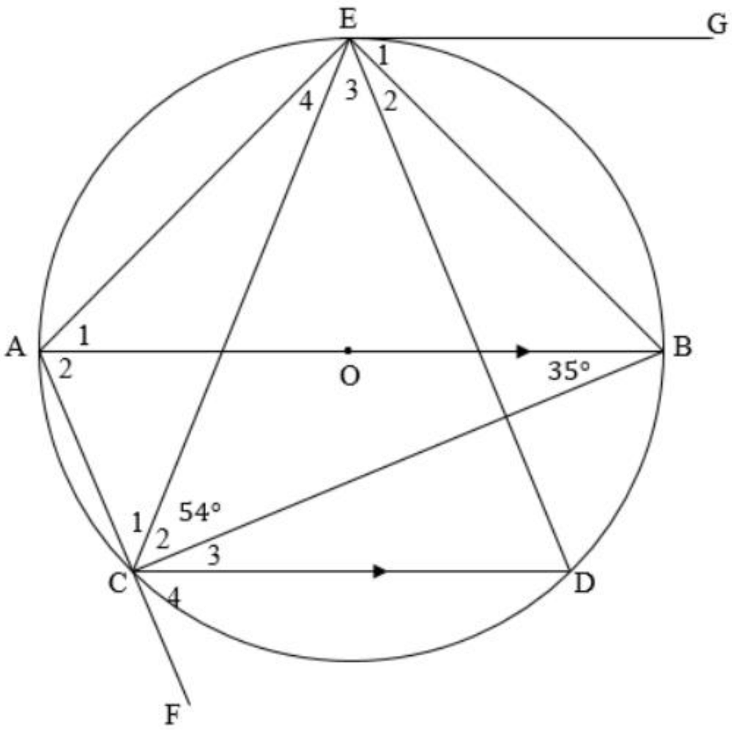
STATEMENT	REASON
7.1. $B\hat{C}D = 90°$	$\angle$ in a semi-circle
7.2 $\hat{A}_1 = A\hat{B}O$ $\hat{A}_1 = 42°$	$\angle$'s opp = sides
7.3 $\hat{O}_2 = \hat{A}_1 + A\hat{B}O$ $\hat{O}_2 = 82°$	Ext $\angle$ of ΔABO
7.4 $\hat{C}_4 = C\hat{B}D$ $\hat{C}_4 = 25°$	Tan-chord

QUESTION 8



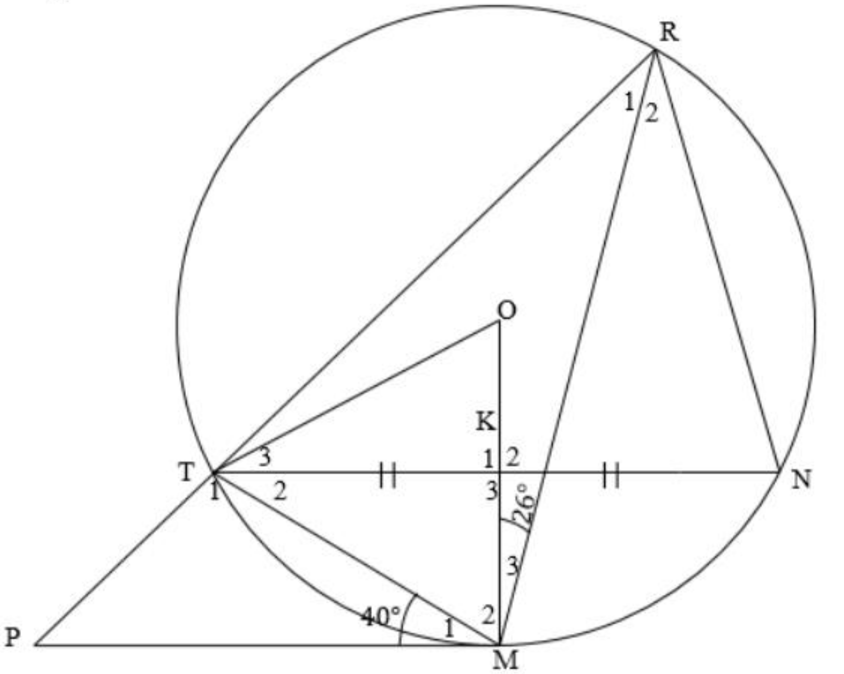
STATEMENT	REASON
8.1 $\hat{A}_1 = \hat{B}_1$ $\hat{A}_1 = 30°$	Tan-chord
8.2 $\hat{T} + \hat{B}_1 = \hat{D}_2$ $\hat{T} = 21°$	Ext ∠ of ΔADT
8.3 $\hat{B}_2 = \hat{T}$ $\hat{B}_2 = 21°$	Alt ∠s, TAN‖CB
8.4 In ΔABD $\hat{A}_2 = 99°$ $\hat{C} + \hat{A}_2 = 180°$ $\hat{C} = 81°$	 Interior ∠s of Δ Opp ∠s of a cyclic quad

QUESTION 9



STATEMENT	REASON
9.1 $\hat{E}_1 = \hat{C}_2$ $\hat{E}_1 = 54°$	Tan-chord
9.2 $A\hat{C}B = 90°$ $\hat{C}_1 = 36°$	$\angle$ in a semi-circle
9.3 $\hat{C}_1 = A\hat{B}C$ $\hat{C}_1 = 35°$	Alt $\angle s$, AB‖CD
9.4 $\hat{E}_2 = \hat{C}_3$ $\hat{E}_2 = 35°$ $A\hat{E}D + \hat{E}_2 = 90°$ $A\hat{E}D = 55°$	$\angle s$ in the same segment $\angle$ in a semi-circle
9.5 $\hat{E}_4 = A\hat{B}C$ $\hat{E}_4 = 35°$ $\hat{E}_3 + \hat{E}_4 = A\hat{E}D$ $\hat{E}_3 + 35° = 55°$ $\hat{E}_3 = 20°$	$\angle s$ in the same segment

QUESTION 10



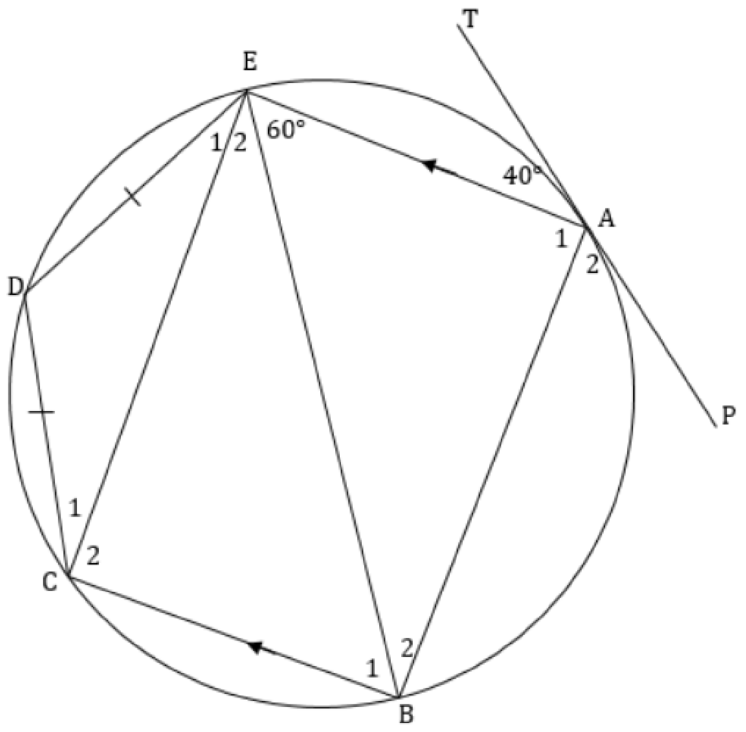
STATEMENT	REASON
10.1. $\hat{M}_1 = \hat{R}_1$ $\hat{R}_1 = 40°$ $T\hat{O}M = 2 \times \hat{R}_1$ $T\hat{O}M = 80°$	Tan-chord $\angle$ at centre $= 2 \times \angle$ at circum
10.2 In ΔTOM $\hat{M}_2 = T\hat{O}M$ $\hat{M}_2 = 50°$ $\hat{N} = R\hat{M}T$	 $\angle s$ opp = sides $\angle s$ in the same seg
$\hat{N} = 76°$	
10.3 In ΔTKM $\hat{T}_2 + \hat{K}_1 + \hat{M}_2 = 180°$ $\hat{T}_2 = 40°$ $\hat{T}_3 + \hat{T}_2 = 50°$ $\hat{T}_3 = 10°$	 Interior $\angle s$ of Δ

QUESTION 11

STATEMENT	REASON
11.1 $\hat{AOB} = 2 \times \hat{D}_1$ $\hat{D}_1 = 65°$	$\angle$ at centre $= 2 \times \angle$ at circum
11.2 $\hat{B}_1 = \hat{D}_1$ $\hat{B}_1 = 65°$	Tan-chord
11.3 $B\hat{A}D = \hat{B}_1$ $B\hat{A}D = 65°$	Alt $\angle s$, AD||EF
11.4 $B\hat{A}D + \hat{C} = 180°$ $\hat{C} = 125°$	Opp $\angle s$ of a cyclic quad
11.5 $D\hat{B}F = \hat{D}_1$ $D\hat{B}F = 65°$ $\hat{B}_3 + D\hat{B}F = 90°$ $\hat{B}_3 = 25°$	Alt $\angle s$, AD||EF Tan $\perp$ rad
11.6 $GD = AG$ $GD + AG = AD$	Line from centre $\perp$ chord

$AG + AG = AD$ $\quad (GD = AG)$ $2GD = \frac{\sqrt{7}}{2}$ $GD = \frac{\sqrt{7}}{4}$	

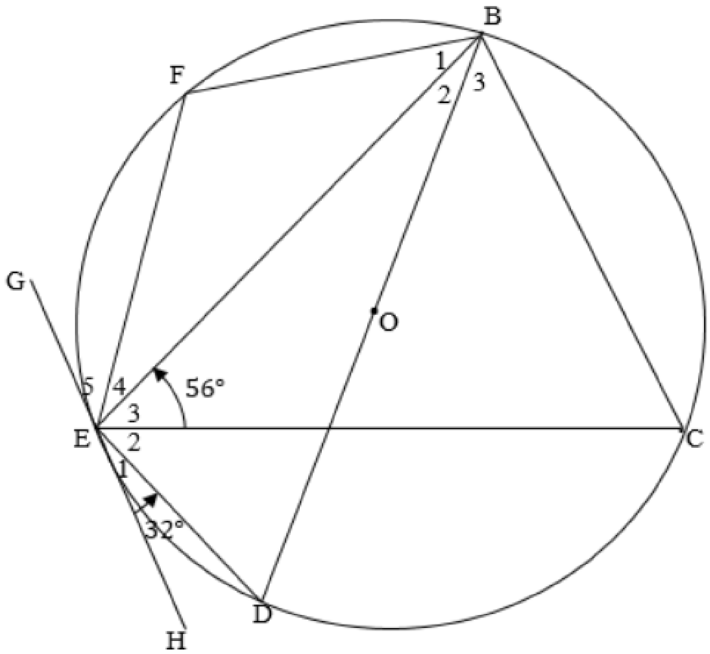
QUESTION 12



STATEMENT	REASON
6.1 BCDE ABCE	
6.2 $\hat{B}_2 = E\hat{A}T$ $\hat{B}_2 = 40°$	Tan-chord
6.3 $\hat{B}_1 = A\hat{E}B$ $\hat{B}_1 = 60°$	Alt $\angle s$, EA‖CB

6.4 $\hat{B}_1 + \hat{D} = 180°$ $\hat{D} = 120°$	Opp $\angle s$ of a cyclic quad
6.5 In ΔCDE $\hat{E}_1 = \hat{C}_1$ $\hat{E}_1 + \hat{C}_1 + \hat{D} = 180°$ $2\hat{E}_1 + 120° = 180° \quad (\hat{E}_1 = \hat{C}_1)$ $\hat{E}_1 = 30°$	Isosceles triangle $\angle s$ opp = sides

QUESTION 13



STATEMENT	REASON
1.1 $\hat{E}_2 + \hat{E}_3 = 90°$ $\hat{E}_2 = 34°$	$\angle$ in a semi-circle

GRADE 12 LEVEL 3 AND 4 SOLUTIONS

QUESTION 1

STATEMENT	REASON
1.1 In ΔPRW $\frac{WT}{RT}=\frac{WS}{SP}$ $\frac{WT}{6}=\frac{2}{3}$ $WT = 4$ cm	Proportionality theorem, ST//PR
1.2 In ΔPQW $\frac{QW}{WR}=\frac{WP}{WS}$ $\frac{QW}{10}=\frac{5}{3}$ $QW = 16{,}67$ cm	Proportionality theorem, RS//PQ

QUESTION 2

STATEMENT	REASON
2.1 In ΔFAC $\frac{AE}{EF}=\frac{AB}{BC}$ $\frac{AE}{EF}=\frac{4}{6}$ $\frac{AE}{EF}=\frac{2}{3}$	Proportionality theorem, BE//CF
In ΔFAD $\frac{CD}{AC}=\frac{EF}{AE}$ $\frac{CD}{10}=\frac{3}{2}$ $CD = 15$ units	Proportionality theorem, EC//FD

2.2 In ΔFAD and In ΔFEC $\frac{\text{Area } \Delta\text{FEC}}{\text{Area } \Delta\text{FEC}}$ $= \frac{\frac{1}{2} \times \text{EF} \times h}{\frac{1}{2} \times \text{AF} \times h}$ $= \frac{\text{EF}}{\text{AF}}$ $= \frac{6}{10}$ $= \frac{3}{5}$	Area of Δs

QUESTION 3

STATEMENT	REASON
3.1 In ΔPTS $\cos P\hat{T}S = \frac{(4,5)^2 + (3,6)^2 - (1,5)^2}{2(4,5 \times 3,6)}$	Cosine rule
$P\hat{T}S = 17,15°$ In ΔQRS $\cos Q\hat{R}S = \frac{(12)^2 + (9,6)^2 - (4)^2}{2(12 \times 9,6)}$	
$Q\hat{R}S = 17,15°$	Cosine rule
$P\hat{T}S = P\hat{R}S$	Both $= 17,15°$
$\therefore$ PT is a tangent to circle passing through T, S and R.	Converse of Tan-chord

3.2 In ΔPTS $\cos P\hat{T}S = \dfrac{(4{,}5)^2 + (1{,}5)^2 - (3{,}6)^2}{2(4{,}5 \times 1{,}5)}$ $P\hat{T}S = 58°$ $\dfrac{TR}{\sin 58°} = \dfrac{4{,}5}{\sin 17{,}15°}$ $TR = 12{,}94$ units $TQ = 3{,}34$ units	Cosine rule Sine rule $TR = TQ + QR$

QUESTION 4

STATEMENT	REASON
4.1 In ΔABP $\dfrac{AS}{SP} = \dfrac{AR}{AB}$ $\dfrac{AS}{AP} = \dfrac{3}{5}$ $\therefore \dfrac{AS}{SC} = \dfrac{3}{7}$	Proportionality theorem, RS//CF
4.2 In ΔCSR $\dfrac{RT}{TC} = \dfrac{SP}{PC}$ $\dfrac{RT}{TC} = \dfrac{2}{5}$	Proportionality theorem, PT//RS

4.3 $\frac{\text{Area of } \Delta RSA}{\text{Area of } \Delta RSC}$ $= \frac{\frac{1}{2} \times AS \times h}{\frac{1}{2} \times SC \times h}$ $= \frac{3}{7}$	
4.4 $\frac{\text{Area of } \Delta TPC}{\text{Area of } \Delta RSC}$ $= \frac{\frac{1}{2} \times PC \times h}{\frac{1}{2} \times SC \times h}$ $= \frac{5}{7}$	

QUESTION 5

STATEMENT	REASON
5.1 In ΔADE $\frac{GD}{GA} = \frac{DF}{FE}$ $\frac{GD}{x+3} = \frac{3}{6}$ $GD = \frac{x+3}{2}$ $CD = \frac{x+3}{2} + x$ $CD = \frac{3x+3}{2}$	 Proportionality theorem, FG//AE

5.2 In ΔADE $\frac{\text{CD}}{\text{AC}} = \frac{\text{BE}}{\text{AB}}$ $\frac{\left(\frac{3x+3}{2}\right)}{3} = \frac{3}{1}$ $\frac{3x+3}{2} = 9$ $x = 5$ units	Proportionality theorem, BC//DE
5.3 In ΔADE $\frac{\text{CD}}{\text{AC}} = \frac{\text{BE}}{\text{AB}}$ $\frac{\text{BC}}{9} = \frac{3}{12}$ $\text{BC} = 2{,}25$ units	Proportionality theorem, BC//DE

5.4 $\frac{\text{Area of } \Delta\text{ABC}}{\text{Area of } \Delta\text{GFD}}$ $= \frac{\frac{1}{2} \times \text{AC} \times h}{\frac{1}{2} \times \text{GD} \times h}$ $= \frac{3}{4}$	

QUESTION 6

STATEMENT	REASON
6.1 In ΔPRA $\frac{PC}{CA} = \frac{PB}{BR}$ $\frac{PC}{CA} = \frac{1}{2}$ In ΔBCQ $\frac{BD}{DQ} = \frac{CA}{AQ}$ $\frac{BD}{DQ} = \frac{1}{2}$ $BD : DQ = 1 : 2$	 Proportionality theorem, BC//AR Proportionality theorem, BC//AD
6.2 $\frac{\text{Area of } \Delta PRA}{\text{Area of } \Delta QRA}$ $= \frac{\frac{1}{2} \times AP \times h}{\frac{1}{2} \times AQ \times h}$ $= \frac{4}{9}$	

6.3 $\frac{\text{Area of } \Delta BQC}{\text{Area of } \Delta RPQ}$ $= \frac{\frac{1}{2} \times QC \times h}{\frac{1}{2} \times AP \times h}$ $= \frac{11}{3}$	

QUESTION 7

STATEMENT	REASON
In ΔABE	
$\frac{BK}{KE} = \frac{BD}{DA}$	Proportionality theorem, DK//AE
$\frac{BK}{KE} = \frac{3}{2}$	
Then:	
$\frac{BE}{EC} = \frac{5}{3}$	Given
In ΔCDK	
$\frac{CP}{PD} = \frac{EC}{EK}$	Proportionality theorem, PE//DK
$\frac{CP}{PD} = \frac{3}{2}$	

QUESTION 8

STATEMENT	REASON
8.1	
$\hat{E}_2 = \hat{E}_1$	$\angle$s opp. = sides, BF = BD
$\hat{B}_2 = \hat{E}_1$	Tan-chord
$\hat{B}_2 = \hat{E}_2$	Both = $\hat{E}_1$
8.2	
$\hat{B}_2 = \hat{E}_2$	Proven
$\hat{D}_1 = \hat{F}$	Ext $\angle$ of a Δ
$\hat{F} = \hat{D}_1$	3rd $\angle$ of a Δ
$\Delta BDA \| \| \Delta EFB$	AAA

8.3	
$\frac{BD}{AD} = \frac{EF}{BF}$	$\Delta BDA \mid\mid\mid \Delta EFB$
$\frac{BD}{AD} = \frac{EF}{BD}$	Given $BD = BF$
$BD^2 = AD.EF$	

QUESTION 9

STATEMENT	REASON
10.1	
$R\hat{N}S = P\hat{N}Q$	Both $= x$
$\hat{R}_1 = \hat{Q}_2$	$\angle's$ in the same seg.
$\hat{S}_1 = N\hat{P}Q$	$3^{rd} \angle$ in a Δ
$\therefore \Delta NSR \mid\mid\mid \Delta NPO$	AAA
10.2	
$N\hat{P}S = N\hat{Q}R$	Both $= \hat{N}_1 + x$
$\hat{P}_1 = \hat{Q}_1$	$\angle's$ in the same seg.
$\hat{S}_2 = \hat{R}_2$	$3^{rd} \angle$ in a Δ
$\therefore \Delta NQR \mid\mid\mid \Delta NPS$	AAA

QUESTION 10

STATEMENT	REASON
11.1	
In ΔABC and in ΔADT	
$\hat{A}_1 = \hat{B}_2$	Tan-chord
$\hat{A}_3 = \hat{B}_2$	Alt $\angle$s, $AC \mid\mid BT$
$\hat{A}_1 = \hat{A}_3$	Both $= \hat{B}_2$
$\hat{T}_3 = \hat{C}$	Ext $\angle$ of a cyclic quad.
$\hat{D}_1 = \hat{B}_1$	$3^{rd} \angle$ in a Δ
$\therefore \Delta ABC \mid\mid\mid \Delta ADT$	AAA

11.2 $\hat{T}_1 = \hat{A}_3$ $\hat{T}_4 = \hat{T}_1$ $\hat{T}_4 = \hat{A}_3$	 $\angle's$ in the same seg. Vert. opp. $\angle's$ Both $= \hat{T}_1$

$\therefore \hat{T}_4 = \hat{A}_1$ $\therefore$ PT is a tangent to a circle passing through A, D and T.	Proven $\hat{A}_1 = \hat{A}_3$ Converse of Tan-chord
11.3 In ΔATP and in ΔTDP $\hat{T}_4 = \hat{A}_1$ $A\hat{P}T$ $\hat{D}_1 = A\hat{T}P$ $\Delta ATP \vert\vert\vert \Delta TDP$	 Tan-chord Common $\angle$ 3rd $\angle$ in a Δ AAA
11.4 $\frac{AP}{PT} = \frac{PT}{PD}$ $PD \times AP = PT^2$ $AD = \frac{2}{3}AP$ $PD = \frac{AP}{3}$ $\frac{AP}{3} \times AP = PT^2$ $AP^2 = 3PT^2$	 $\Delta ATP \vert\vert\vert \Delta TDP$ Given

QUESTION 11

STATEMENT	REASON
14.1	
$\hat{W}_2 = \hat{U}_2$	Alt ∠s, WT‖SU
$\hat{W}_2 = \hat{P}_2$	∠s opp. = sides, WU = TU
$\hat{U}_2 = \hat{P}_2$	Both = $\hat{W}_2$
∴ SU is a tangent to a circle passing through P, W, U and T.	Converse of Tan-chord
14.2 In ΔSPU and in ΔSUW	
$\hat{S}$	Common ∠
$\hat{U}_2 = \hat{P}_2$	Tan-chord
$\hat{W}_3 = S\hat{U}P$	3rd ∠ in a Δ
∴ $\Delta SPU \parallel\mid \Delta SUW$	AAA

QUESTION 12

STATEMENT	REASON
13.1 In ΔAPC and in ΔABP	
$\hat{A}$	Common ∠
$\hat{P}_1 = \hat{C}_2$	Tan-chord
$\hat{B}_2 = A\hat{P}C$	3rd ∠ in a Δ
$\Delta APC \parallel\mid \Delta ABP$	AAA
13.2 $\frac{AP}{AC} = \frac{AB}{AP}$ $AP^2 = AB \times AC$	$\Delta APC \parallel\mid \Delta ABP$

13.3 In ΔAPC and in ΔCDP	
$\hat{P}_1 = \hat{C}_2$	Alt ∠s, BC‖DP
in ΔCDP	
$\hat{C}_1 = 180° - (x + y)$	Sum of int. ∠s in a Δ
$\hat{B}_2 = \hat{D}$	Ext. ∠ of a cyclic quad
$\hat{B}_2 = x$	
in ΔABP	
$\hat{A} = 180° - (x + y)$	Sum of int. ∠s in a Δ
$\hat{C}_1 = \hat{A}$	Both $180° - (x + y)$
$\hat{D} = A\hat{P}C$	3rd ∠ in a Δ
$\Delta APC \| \| \Delta CDP$	AAA

QUESTION 13

STATEMENT	REASON
15.1.1 In ΔPRT	
$\frac{PK}{PT} = \frac{1}{3}$	Given: PK : KT = 1 : 2
$\frac{PQ}{PR} = \frac{\sqrt{2}}{\sqrt{18}}$	Given: $PR = \sqrt{18}$ and $PQ = \sqrt{2}$
$\frac{PQ}{PR} = \frac{1}{3}$	
$\frac{PK}{PT} = \frac{PQ}{PR}$	Both $= \frac{1}{3}$
$\therefore RT \| \| QK$	Converse of Line ‖ 3rd side

15.1.2 $S\hat{T}P = 90°$ $\hat{R}_1 = 90°$ $\hat{Q}_1 = \hat{R}_1$ $\hat{Q}_1 = S\hat{T}P$ $\therefore$ TKQS is a cyclic quadrilateral	 $Tan \perp r$ $\angle$ in a semi-circle Corr. $\angle$s, QK||RT Both = 90° Converse of Ext. $\angle$ of a cyclic quad, $\hat{Q}_1 = S\hat{T}P$
15.1.3 In ΔQRT and in ΔKTS $\hat{Q}_1 = S\hat{T}P$ $\hat{Q}_3 = \hat{K}_1$ $\hat{S}_2 = \hat{T}_2$ $\therefore \Delta QRT \| \| \Delta KTS$	 Both = 90° $\angle's$ in the same seg. 3rd $\angle$ in a Δ AAA

QUESTION 14

STATEMENT	REASON
14.1 $L\hat{N}T = 90°$ $L\hat{N}T + L\hat{P}T = 180°$ $\therefore$ TKQS is a cyclic quadrilateral	 $\angle$ in a semi-circle Converse of opp. $\angle$s of a cyclic quad, $L\hat{N}T + L\hat{P}T = 180°$
14.2 In ΔKLN $\hat{K} = 90° - x$ $\hat{N}_1 = \hat{K}$ $\hat{N}_1 = 90° - x$	 Sum of int. $\angle$s in a Δ Tan-chord

14.3	
$K\hat{P}T = L\hat{N}T$	Ext. ∠ of a cyclic quad TPLN
$\hat{K}$	Common ∠
$\hat{L} = \hat{T}_2$	3^{rd} ∠ in a Δ
$\therefore \Delta KTP \parallel\mid \Delta KLN$	AAA

QUESTION 15

STATEMENT	REASON
15.1	
$\hat{V}_3 = \hat{S}_2$	Tans from same point
$\hat{V}_3 = x$	
$\hat{P} = \hat{S}_2$	Tan-chord
$\hat{P} = x$	
$\hat{W}_3 = \hat{P}$	Corr. ∠s, WT‖PV
15.4	
$\hat{W}_3 = \hat{V}_3$	Both $= x$
∴ WSTV is a cyclic quadrilateral	Converse of ∠s in the same seg.
15.3	
$\hat{W}_2 = \hat{S}_2$	∠s in the same seg.
$\hat{V}_1 = \hat{W}_2$	Alt. ∠s, WT‖PV
$\hat{W}_2 = x$	
$\hat{P} = x$	Proven
∴ WPV is isosceles	$\hat{V}_1 = \hat{P}$, both $= x$
15.4	
$\hat{P} = \hat{S}_2$	Tan-chord
$\hat{V}_1 = \hat{V}_3$	Both $= x$, proven
$\hat{W}_1 = \hat{P}$	3^{rd} ∠ in a Δ
∴ ΔWPV‖‖ΔTSV	AAA